U.S. Patent Classification System - Classification Definitions as of December 31, 1999

EXPLANATION OF DATA

(definitions have been obtained from the *Patents ASSIST* CD-ROM which is produced by the U.S. Patent and Trademark Office Optical Disc Publishing Program)

Class 705

DATA PROCESSING: FINANCIAL, BUSINESS PRACTICE, MANAGEMENT, OR COST/PRICE DETERMINATION

Class Definition:
This is the generic class for apparatus and corresponding methods for performing data processing operations, in which there is a significant change in the data or for performing calculation operations wherein the apparatus or method is uniquely designed for or utilized in the practice, administration, or management of an enterprise, or in the processing of financial data.
This class also provides for apparatus and corresponding methods for performing data processing or calculating operations in which a charge for goods or services is determined.
SCOPE OF THE CLASS
1. The arrangements in this class are generally used for problems relating to administration of an organization, commodities or financial transactions.
2. Mere designation of an arrangement as a "business machine" or a document as a "business form" or "business chart" without any particular business function will not cause classification in this class or its subclasses.
3. For classification herein, there must be significant claim recitation of the data processing system or calculating computer and only nominal claim recitation of any external art environment. Significantly claimed apparatus external to this class, claimed in combination with apparatus under the class definition, which perform data processing or calculation operations are classified in the class appropriate to the external device unless specifically
excluded therefrom.
4. Nominally claimed apparatus external to this class in combination with apparatus under the class definition is classified in this class unless provided for in the appropriate external class.
5. In view of the nature of the subject matter included herein, consideration of the classification schedule for the diverse art or environment is necessary for proper search.

REFERENCES TO OTHER CLASSES

SEE OR SEARCH CLASS:
177, Weighing Scales, 25.11 for a computerized scale.
186, Merchandising, various subclasses for customer service methods and apparatus in a variety of

areas including banking, restaurant and stores.
235, Registers, various subclasses for basic machines and associated indicating mechanisms for ascertaining the number of movements of various devices and machines, plus machines made from these basic machines alone (e.g., cash registers, voting machines), and in combination with various perfecting features, such as printers and recording means. In addition, search Class 235 for various data bearing record controlled systems. Search subclasses 375-386 for a system having a detail of a record-sensing device in combination with a system utilized for banking, determining credit, maintaining an inventory, access control, vending, voting, time or operations analysis and having no more than a nominal recitation of a computer or data processing arrangement. Search subclasses 7+ for cash register; and subclass 61 for mechanically computing a cost/price ratio. Note that a nominally claimed record or card sensor is considered to be a peripheral of the data processing system.
283, Printed Matter, various subclasses for business forms and methods of using such forms.
307, Electrical Transmission or Interconnection Systems, various subclasses for generic residual electrical transmission or interconnection systems and miscellaneous circuits.
340, Communications: Electrical, various subclasses for residual electrical communication systems, 825.30 for communication details including authorization, vending, credit and access control; and see related classes elsewhere.
341, Coded Data Generation or Conversion, various subclasses for electrical pulse and digital code conversion.
345, Computer Graphics Processing, Operator Interface
Processing and Selective Visual Display Systems, subclasses 418-475 for data presentation/computer s:graphics processing; subclasses 112 through 144 for data presentation processing; and subclasses 1 through 111 for the selective control of two or more light generating or light controlling display elements in accordance with a received image signal.
359, Optics: Systems (including Communications) and Elements, 107 for an optical computing arrangement.
360, Dynamic Magnetic Information Storage or Retrieval, which is an integral part of Class 369 following subclass 18, for record carriers and systems wherein information is stored and retrieved by interaction with a medium and there is relative motion between a medium and a transducer, for example, magnetic disk drive devices and control thereof, per se.
365, Static Information Storage and Retrieval, various subclasses for addressable static singular storage elements or plural singular storage elements of the same type.
369, Dynamic Information Storage or Retrieval, various subclasses for record carriers and systems wherein information is stored and retrieved by interaction with a medium and there is relative motion between a medium and a transducer.
370, Multiplex Communications, various subclasses for generic multiplexing and demultiplexing systems.
371, Error Detection/Correction and Fault Detection/Recovery, various subclasses for generic electrical pulse or pulse coded data error detection and correction.
375, Pulse or Digital Communications, various subclasses for generic pulse or digital communication systems.
377, Electrical Pulse Counters, Pulse Dividers, or Shift Registers: Circuits and Systems, various subclasses for generic circuits for pulse counting.
379, Telephonic Communications, various subclasses for two-way electrical communication of intelligible audio information of arbitrary content over a link including an electrical conductor. In addition, search 112 for a computer controlled telephone charge determining arrangement; subclass 284 for a processor controlled central switching arrangement.
380, Cryptography, 3 for stored information access or copy prevention (e.g., software program protection or computer virus detection) in combination with data encryption, and subclasses 22 through 25 for electric signal modification with record carrier, computer, electronic funds transfer, user or record actuated authentication.
382, Image Analysis, appropriate subclasses for operations
performed on image data with the aim of measuring a characteristic of an image, detecting variations, detecting structures, or transforming the image data, and for procedures for analyzing and categorizing patterns present in image data.
395, Information Processing System Organization, subclasses 180+ for reliability and availability in a

digital data processing system; subclasses 200.01+ multicomputer data transferring.
434, Education and Demonstration, subclasses 107-110, 219+ and 306 for education or demonstration of business or economics, occupations and voting, subclasses 322+ for question or problem eliciting response.
463, Amusement Devices: Games, 1, when there is a recitation (according to paragraph 3, under Scope of the Class, in the Class Definition of this class (705)) of a method or apparatus for moving or processing information specified as game or contest information, especially subclasses 16+, where game or contest information relates to a chance-type game (i.e., one that involves an award or prize based upon the occurrence of a chance happening or event; e.g., lottery, keno, slot machine, etc.). However, in order for the specified information to be considered significant for placement in Class 463, there must be significant game processing. For example, the mere printing of a selected lottery number is not significant unless there is also included an element of a game for Class 463 (e.g., a determination of whether the number is a winning number, such as by lot matching, a determination of an award or prize value associated with a number, etc.). A "coupon" or price adjustment given to a patron based upon an item purchased or another predetermined or nonrandom criteria is not considered to be a chance event or happening appropriate for Class 463.
700, Data Processing: Generic Control Systems or Specific Applications, subclasses 90-306 for devices or methods for controlling the processing or manufacturing of, or being responsive to a physical or mechanical condition of, a product or material, and. subclasses 245-264 for robot control.
704, Data Processing: Speech Signal Processing, Linguistics, Language Translation, and Audio Compression/Decompression, 231 for a speech recognition system.
708, Electrical Computers: Arithmetic Processing and Calculating, 100 for electrical digital calculating computer combined with diverse art device such as a checkbook or calendar or with inputs or outputs specialized for a particular environment including business.
902, Electronic Funds Transfer, for art collections including a detail of a security measure, an ATM machine, a terminal or an identifier used in an electronic funds transfer.

GLOSSARY:
ARRANGEMENT
Either a device or a method of use of a device for performing the indicated process.
CALCULATING OPERATIONS
Arithmetic or some limited logic operations performed upon or with signals representing numbers or values.
COMPUTER
A machine that inputs data, processes data, stores data, and outputs data.
DATA
Representation of information in a coded manner suitable for communication, interpretation, or processing.
DATA PROCESSING
(For the purpose of this class) A systematic operation on data in accordance with a set of rules which results in a significant change in the data.
DEVICE
An assemblage of components at a single location or which may have its several components at geos:graphically distinct locations, i.e., a network.
ENTERPRISE
A conventional business organization, a governmental organization or a nonprofit organization.
PRACTICE
A function directly related to the commercial activity of an enterprise (e.g. the exchange, buying or selling of commodities).

SUBCLASSES

Subclass: 1
AUTOMATED ELECTRICAL FINANCIAL OR BUSINESS PRACTICE OR MANAGEMENT ARRANGEMENT:
Under the class definition. Subject matter wherein an electrical apparatus and its corresponding

arrangement for determining or submitting a tax or tax form to a governmental entity.
Subclass: 32
Time accounting (time and attendance, monitoring billable hours):
Under subclass 30. Subject matter for analysis or allocation of hours either (1) worked by an individual, or (2) utilized by and billable to an individual or other entity.

SEE OR SEARCH THIS CLASS, SUBCLASS:
11 for job performance analysis, computing quantities additional to attendance (e.g., productivity).

SEE OR SEARCH CLASS:
364, Electrical Computers and Data Processing Systems, subclass 143 for time responsive control in a plural processor controller arrangement.
368, Horology, Time Measuring Systems or Devices, appropriate subclasses for time responsive control of general utility.
Subclass: 33
Checkbook balancing, updating or printing arrangement:
Under subclass 30. Subject matter including an arrangement for attaching a data processing device to a checkbook, which processes transaction data to verify, or carry forward, the checkbook balance, or to print a check, upon to entry of an account transaction.

SEE OR SEARCH CLASS:
708, Electrical Computers: Arithmetic Processing and Calculating, subclass 106 for a checkbook attached digital calculator, in which the transaction information is limited to the date or transaction amount.
Subclass: 34
Bill preparation:
Under subclass 30. Subject matter including an arrangement for generating a notice of payment due.
Subclass: 35
Finance (e.g., banking, investment or credit):
Under subclass 1. Subject matter drawn to a computerized arrangement for planning the disposition or use of funds or securities, or extension of credit.

SEE OR SEARCH CLASS:
186, Merchandising, for a banking structural arrangement.
235, Registers, subclass 379 for record-sensing devices in combination with a system that maintain financial accounts; i.e., a banking system; and subclass 380 for a system that includes the ascertaining of credit.
Subclass: 36
Portfolio selection, planning or analysis:
Under subclass 35. Subject matter drawn to a computerized arrangement for planning the selection or evaluation of securities or other investments for a single entity.
(1) Note. The term "entity" refers to an individual or other legally recognized body.
Subclass: 37
Trading, matching, or bidding:
Under subclass 35. Subject matter including the trading or exchange of securities or commodities within an organized system.
(1) Note. This subclass includes distribution of services or products (e.g., utilities, heating, etc.) in a building by an "auction" or bidding system.
Subclass: 38
Credit (risk) processing or loan processing (e.g.,mortgage):
Under subclass 35. Subject matter drawn to a computerized arrangement for evaluation of the risk factors in a loan determination.

SEE OR SEARCH CLASS:
235, Registers, subclass 380 for a record-sensing device in combination with a system that includes the ascertaining of credit.
Subclass: 39

United States Patent *5,812,987*
Luskin , et al. **September 22, 1998**

Investment fund management method and system with dynamic risk adjusted allocation of assets

Abstract

An invention for managing assets in one or more investment funds over a specified time. A fund comprises a plurality of assets (e.g., stocks bonds, currencies, gold, silver, oil, gas). A time horizon H.sub.t representing the expected date at which cash will be withdrawn from the fund is associated with each fund. A time L.sub.H represents the length of time remaining between the present time and the horizon time H.sub.t. A risk tolerance, R.sub.I, changes as a function of the decreasing time to horizon L.sub.H. Typically, the risk tolerance decreases as the fund approaches the time horizon H.sub.t (e.g., investments become more conservative toward the end of the life of the fund). A strategic investment mix of assets in the fund is periodically determined as a function of the changing risk, R.sub.I. Investment modifications are accordingly made in the mix of assets in the fund. In one embodiment, a fund also includes a tactical investment strategy component (e.g., representing 25% of the overall investment strategy). The tactical investment strategy is based on the strategic investment mix. Typically, the percent of strategic investments directed to equity-type assets is used to define the percent of tactical investment that is directed to a first tactical investment allocation strategy. The remaining tactical investment amount is directed to a second tactical investment allocation strategy.

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U.S. Class: **705/36**
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Field of Search: 395/236,237,235 705/36,37,35

References Cited [Referenced By]

U.S. Patent Documents

4752877	Jun., 1988	Roberts et al.	395/235.
4953085	Aug., 1990	Atkins	395/236.
5101353	Mar., 1992	Lupien et al.	395/237.
5126936	Jun., 1992	Champion et al.	395/236.

Other References

"Moneyfor Mutual Funds", Direct Mail Packet, Moneyfor Management, Inc., New Jersey, May 8, 1989.
"Moneyfor Broker Dealer Owner Participation Executive Summary", Moneyfor Management, Inc., New Jersey, 1989.
Kleinholz, Lisa, "Can Software help you get rich?", Home Office Computing vol. 8, No. 8, pp. 30-32, Aug. 1990.

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Parent Case Text

CROSS-REFERENCE TO RELATED APPLICATION

The application is a continuation-in-part of U.S. patent application Ser. No. 08/108,438, "Investment Fund Management Method and System," by Luskin et al., filed Aug. 18, 1993 now abandoned, which subject matter is incorporated herein by reference.

Claims

1. A computer system for managing assets in each of a plurality of investment funds, the system comprising:

a processor for executing programmed instructions and for storing and retrieving data;

program memory, coupled to the processor, for storing program instructions for execution by the processor;

an output device, coupled to the processor, for displaying data;

an input device, coupled to the processor, for accepting input data associated with each investment fund for storage in the memory, including:

a time horizon for each investment fund, and

an actual investment mix among the assets in each investment fund; and

an investment program, stored in the memory and executable on the processor, for automatically and periodically:

determining for each investment fund a current risk level for the investment fund as a function of the time horizon and a current date, the current risk level determined by: ##EQU1## where R.sub.I is the current risk level; L.sub.H is a length of time to the time horizon;

F is a constant risk factor selected for a predetermined maximum amount of risk; and

T.sub.1 and T.sub.2 are times;

determining a risk adjusted asset mix for each investment fund as a function of the current risk level, and

modifying the investment mix of each investment fund as a function of the risk adjusted asset mix.

2. The system of claim 1, wherein the asset mix is a strategic asset mix limited to assets for markets assumed to be in equilibrium, further comprising:

a tactical investment program, stored in the memory and executable on the processor, for:

determining a tactical investment mix for the fund as a function of the strategic asset mix, the tactical investment mix consisting of assets for markets assumed to not be in equilibrium; and

modifying the investment mix of the fund as a function of the tactical investment mix.

3. A computer implemented method for managing assets in an investment fund using a computer comprising a processor, storage, and a memory, the method comprising the steps of:

establishing, via the processor, an investment fund, the investment fund having an actual asset allocation including a plurality of assets, a time horizon defining a maturity date for the investment fund, each asset in the asset allocation being a member of an asset class, each asset class having an asset class weight;

periodically determining, via the processor, a current risk level for the investment fund as a function of the time horizon and a current date, the current risk level determined by: ##EQU2## ##EQU3## where R.sub.I is the current risk level; L.sub.H is a length of time to the time horizon;

F is a constant risk factor selected for a predetermined maximum amount of risk; and

T.sub.1 and T.sub.2 are times;

periodically determining, via the processor, a risk adjusted asset allocation for the investment fund by determining a current asset weight for each asset class in the asset allocation as a function of the current risk level; and

purchasing or disposing of assets in each asset class of the fund to match the actual asset allocation of the investment fund to the risk adjusted asset allocation of the investment fund.

4. The method of claim 3 wherein the asset classes include equity asset classes and income asset classes, and determining the current risk level further comprises:

determining the current risk level as a function of a risk factor, the risk factor allocating a majority of the assets in the risk adjusted asset allocation to the equity asset classes when the time horizon is greater than a first predetermined amount of time, and allocating a majority of the assets in the risk adjusted asset allocation to the income asset classes when the time horizon is less than a second predetermined amount of time, where the first predetermined amount of time is greater than the second predetermined amount of time.

5. The method of claim 3, wherein the step of periodically determining, via the processor, a risk adjusted asset allocation further comprises:

constraining the asset weight of at least one asset class to being less than a predetermined percentage of a total of the asset weights for all asset classes in risk adjusted asset allocation.

6. The computer-implemented method of claim 3 wherein there are a plurality of investment funds, the method further comprising:

periodically and regularly repeating all steps for each investment fund.

7. The computer-implemented method of claim 3 further comprising the steps of:

allocating a first portion of the actual asset allocation to a strategic investment component limited to asset classes for markets assumed to be in equilibrium;

allocating a second, remaining portion of the actual asset allocation to a tactical investment component limited to asset classes for markets assumed to not be in equilibrium, the tactical investment component having at least an equity asset allocation and an income asset allocation;

determining, via the processor, an adjusted tactical investment allocation within the tactical investment component by:

allocating to the equity asset allocation a first portion of the tactical investment component corresponding to a portion of the strategic investment component allocated to equity asset classes; and

allocating to the income asset allocation a second portion of the tactical investment component corresponding to a portion of the strategic investment component allocated to income asset classes; and

purchasing or disposing of assets in the tactical investment component to match the adjusted tactical investment allocation.

8. The computer-implemented method of claim 7 wherein there are a plurality of investment funds, the method further comprising:

periodically and regularly repeating all steps for each investment fund.

9. The method of claim 7 wherein the asset classes include equity asset classes and income asset classes, and the step of determining the current risk level further comprises:

determining the current risk level as a function of a risk factor, the risk factor allocating a majority of the assets in the strategic investment component to the equity asset classes when the time horizon is greater than a first predetermined amount of time, and allocating a majority of the assets in the strategic investment component to the income asset classes when the time horizon is less than a second predetermined amount of time, where the first predetermined amount of time is greater than the second predetermined amount of time.

10. The computer-implemented method of claim 7 wherein the step of determining a tactical investment allocation comprises:

allocating E percent of the assets of the tactical investment component to the equity asset allocation, where E is equal to a percent of the strategic asset allocation allocated to equity asset classes; and

allocating (1-E) percent of the assets of the tactical investment component to the income asset allocation.

11. The computer-implemented method of claim 3 further comprising the steps of:

distributing a second, remaining portion of the assets to a tactical investment component limited to asset classes for markets assumed to not be in equilibrium, the tactical investment component having at least an equity asset allocation and an income asset allocation;

determining, via the processor, an adjusted tactical investment allocation for the fund by:

allocating to the equity asset allocation E percent of the tactical investment component corresponding to a percent of the strategic investment component allocated to equity asset classes; and

allocating to the income asset allocation (1-E) percent of the tactical investment component to the

income asset allocation;

modifying, via the processor, the investment mix of the fund as a function of the tactical investment mix; and

purchasing or disposing of assets in the tactical investment component to match the adjusted tactical investment allocation.

12. A computer implemented method for managing a plurality of investment funds, each investment fund having a time horizon defining a maturity of the investment fund, and an actual investment allocation of the assets in the investment fund, the method comprising:

automatically and periodically determining for each investment fund a risk level for the investment fund as a function of its time horizon and a current date, the risk level determined by: ##EQU4## ##EQU5## where R.sub.I is the risk level; L.sub.H is a length of time to the time horizon;

F is a constant risk factor selected for a predetermined maximum amount of risk; and

T.sub.1 and T.sub.2 are times;

automatically and periodically computing a risk adjusted asset allocation for the assets of each investment fund as a function of the risk level; and

modifying the actual investment allocation of each investment fund to match the risk adjusted asset allocation for the investment fund.

13. The method of claim 12, wherein the step of automatically and periodically computing a risk adjusted asset allocation for each investment fund further comprises:

automatically and periodically computing a strategic asset allocation for each investment fund as a function of the risk level, the strategic asset allocation consisting of a first type of assets of the actual investment allocation in markets assumed to be in equilibrium;

automatically and periodically determining a tactical investment allocation for each investment fund as a function of the strategic asset allocation, the tactical investment allocation consisting of a second type of assets in the actual investment allocation in markets assumed to not be in equilibrium; and

modifying the actual investment allocation, of the fund by purchasing or disposing of first type assets to match the strategic investment allocation and purchasing or disposing of second type assets to match the tactical investment allocation.

14. The method of claim 13, further comprising:

determining the risk level as a function of a risk factor, the risk factor causing the strategic investment allocation to allocate a majority of the assets to the equity asset classes when the time horizon is greater than a first predetermined amount of time, and causing the strategic investment allocation to allocate a majority of the assets to the income asset classes when the time horizon is less than a second predetermined amount of time.

15. In a computer system, including a processor and a memory, an investment program stored in the memory and executable by the processor for managing assets in each of a plurality of investment funds stored in the system, the investment program comprising:

an investment program that accepts a time horizon for each investment fund and an actual investment allocation among the assets in each investment fund, and that automatically and periodically:

determines for each investment fund a risk level for the investment fund as a function of the time

horizon and a current date the risk level determined by: ##EQU6## where R.sub.I is the risk level; L.sub.H is a length of time to the time horizon;

F is a constant risk factor selected for a predetermined maximum amount of risk; and

T.sub.1 and T.sub.2 are times;

determines a risk adjusted asset allocation for each investment fund as a function of the risk level; and

modifies the actual investment allocation of each investment fund to match the risk adjusted asset allocation.

16. A computer implemented method for managing an investment fund having a time horizon defining a maturity of the investment fund, and an actual investment allocation of the assets in the investment fund, the method comprising:

automatically and periodically determining a risk level for the investment fund as a function of the time horizon and a current date, the risk level determined by: ##EQU7## where R.sub.I is the risk level; L.sub.H is a length of time to the time horizon;

F is a constant risk factor selected for a predetermined maximum amount of risk; and

T.sub.1 and T.sub.2 are times;

automatically and periodically computing a risk adjusted asset allocation for the assets of the investment fund as a function of the risk level; and

modifying the actual investment allocation of the investment fund to match the risk adjusted asset allocation for the investment fund.

Description

BACKGROUND OF THE INVENTION

1. Field of the Invention

The present invention relates generally to computer management systems, and specifically to the field of investment fund management systems.

2. Background of the Invention

Long-term investment plans, such as pension plans, enable an investor to save for retirement. Pension plans typically are divided into two types: defined benefit plans; and defined contribution plans. A defined benefit plan involves a promise made by an employer of a lifetime definite benefit paid to the recipient after retirement. The employer funds the promise by investing in trust for the recipient. In a defined contribution plan, the employer and/or the employees contribute a definite amount of money to an investment plan. The benefit paid after retirement is uncertain; it is determined by the performance of the investment plan. 401K plans are an example of defined contribution plans.

In a typical defined contribution plan, the employer establishes a group of investment funds of specified characteristics, from which the employee may select. The group of funds typically is diverse, including funds specializing in bonds, stocks, money markets, currencies, cash, gold, silver, oil, gas, or other precious metals and minerals, and other asset classes, or combinations of asset classes. These funds may be collective investment funds or mutual funds, managed by the employer or by a third party.

An employee typically invests in a fund having characteristics matching his or her investment preferences, such as high-risk/high expected return or low-risk/low expected return. Over time, as the employee's investment preferences change, that employee may move his or her investment from one fund to another to reflect a change in preferences. But because the typical employee generally is not a trained investment expert, it is possible that sub-optimal selections may be made from among the funds, and that the selections are not optimally revised over time.

Many employees in defined contribution plans do not appreciate that investments with high-risk and high expected return--such as equity securities--may be appropriate even for risk averse investors if their time horizons are sufficiently far in the future, and if the investment is properly diversified. Thus, many employees tend to invest more conservatively than an investment expert would do under like circumstances. Because conservative investments generally have low returns over the long term, an unnecessarily risk-averse strategy may be expected to produce disappointing long-term performance.

Many employees in defined contribution plans do not possess sufficient expertise to select investment funds to match their risk preferences, even when their risk preferences are appropriate. As a result, investments may be selected that either expose an employee to unexpected risks, or expose the employee to unexpectedly low returns.

Finally, many employees in defined contribution plans do not possess sufficient expertise, or wish to devote sufficient time and attention, to appropriately revise their selection of funds as market conditions change, and as their own life circumstances change. If an employee neglects to revise his or her investments, or revises them inappropriately--perhaps due to emotions of fear or greed--that employee will be exposed to unexpected and inappropriate risks when market conditions change, or when his or her own life circumstances change.

The typical employee generally is unwilling to pay the costs to obtain private professional investment advice, or may be unaware that it is available. The typical employer generally is unwilling to provide advice to employees, either due to a lack of sufficient expertise, or due to an unwillingness to bear potential legal liabilities. Thus, there remains a need for a system and method for a typical employee in a defined contribution plan to make appropriate investments, to reflect appropriate long-term trade-offs of risk and return, to select investments that accurately reflect those trade-offs, and to revise those investments through time in response to changing market conditions and the employee's changing preferences.

SUMMARY OF THE INVENTION

The present invention relates to a method and system for managing assets in one or more funds over a specified life of the fund. A fund F.sub.n has associated therewith a plurality of assets. Assets may be, for example, bonds, stocks, money markets, cash, gold, silver, oil, gas, other precious metals and minerals, and the like. A time horizon H.sub.t is associated with each fund F.sub.n. The time horizon H.sub.t defines the expected date at which cash may need to be withdrawn from the fund and has an associated parameter L.sub.H representative of the length of time remaining between the present and the time horizon H.sub.t. The investment mix of a fund is strategically adjusted at periodic intervals in accordance with some criteria that are related to the diminishing length to time horizon L.sub.H. The criteria may include a risk level R.sub.I that changes as a function of the remaining length to time horizon L.sub.H associated with each fund. Typically, the risk value decreases as the fund approaches the time horizon H.sub.t (e.g., investments become more conservative towards the end of the life of the fund).

In one embodiment, a fund has both a strategic investment strategy, as described above, and a tactical investment strategy. In a preferred embodiment, 75% of the investment strategy (e.g., cash, 401K, or other investments) is based on the strategic investment component, and 25% is based on the tactical investment component. The tactical investment strategy is based on the strategic investment mix. Typically, the percent of strategic investments directed to equity-type assets is used to define the

percent of tactical investment that is directed to a first tactical investment allocation strategy (e.g., as defined by the Wells Fargo Tactical Asset Allocation). The percent of strategic investments directed to non-equity-type assets is used to define the percent of tactical investment that is directed to a second tactical investment allocation strategy (e.g., as defined by the Wells Fargo U.S. Treasury Asset Allocation).

The asset management system and method of the present invention is implemented in a conventional computer system comprising a processor, storage,. and memory. A software program implemented in the memory and executed on the processor of the computer controls the asset management system. Adjustments in the mix of the fund involve physically selling or buying tangible assets (e.g., stocks, bonds, paper, currencies, gold, silver, precious metals and minerals, oil, gas) on the open market.

BRIEF DESCRIPTION OF THE FIGURES

FIG. 1 illustrates an exemplary investment fund structure embodying the present invention.

FIG. 2 illustrates a shift in distribution of cash among the portfolios of an exemplary fund, over the life of the fund, in accordance with the present invention.

FIG. 3 is a block diagram of an exemplary data processing system embodying the present invention.

FIG. 4 is a flow chart illustrating one embodiment of the present invention.

FIGS. 5A-5D are return-to-risk charts of exemplary funds managed in accordance with the present invention.

FIG. 6 is a flow chart illustrating one embodiment of the present invention.

FIG. 7 is a high-level flow chart illustrating one embodiment of the investment management method of the present invention.

FIG. 8 is a flow chart of the determination of strategic asset class weights.

FIG. 9 is a flow chart of the determination of tactical investments.

DESCRIPTION OF THE PREFERRED EMBODIMENTS

The present invention is a system and method for managing assets in investment funds wherein each fund has a fixed time horizon H.sub.t and a diminishing length to horizon L.sub.H. That is, the fund is managed by manipulating the investment mix of the fund in accordance with some criteria related to the diminishing length to horizon L.sub.H of the fund.

Generally, the method includes establishing an investment fund F.sub.n with an associated fixed time horizon H.sub.t and a diminishing length to horizon L.sub.H. A relationship is established between at least one investment asset and the fund in accordance with a first set of criteria. Assets may be, for example, stocks, bonds, paper, currencies, gold, silver, precious metals and minerals, oil, gas and the like. Such tangible assets are physically bought and possessed, and/or sold and disposed of on the open market. One or more assets may be associated with a portfolio, which portfolio may be in some relationship with the fund. Cash may be directed to selected ones of the assets or portfolios to establish an investment mix for the investment fund. Cash, as used herein, may refer to any assets of value. According to the invention, the investment mix is adjusted as a function of the diminishing length to horizon L.sub.H of the investment fund F.sub.n. Thus, as the investment fund matures, the investment mix is changed.

One exemplary manner in which the investment mix may change over the life of the fund is to change the distribution of new cash across the portfolios as such cash is invested in the fund or otherwise becomes available. Another manner is to change the asset or portfolio mix by exchanging

assets or portfolios having one type of characteristic for assets or portfolios having a different type of characteristic. Typically, the investment strategy of an investment fund will become more conservative as it approaches maturity and the distribution of cash among the assets or the mix of portfolios reflects that change in the fund.

FIG. 1 graphically depicts an exemplary investment fund structure embodying the present invention. As shown, a plurality of investment funds F.sub.n are made available to individual investors. Each fund has an associated time horizon H.sub.t, which indicates the date that the individual investor anticipates receiving money from the fund. The time horizon H.sub.t preferably is expressed in terms of a specific calendar date for maturity. Also associated with each fund F.sub.n is a length to horizon L.sub.H which represents the remaining time between the present and the time horizon H.sub.t.

In an alternative embodiment, each fund F.sub.n also may have an associated risk value R, representative of whether the fund generally represents a conservative or aggressive investment strategy. The risk R of a fund F.sub.n provides individual investors with information about the fund to assist tailoring their investments in accordance with their own risk preferences. In another embodiment, an individual investor may impose other constraints for a particular fund, such as designation of allowable investments. For example, an investor may designate that an investment should consist only of domestic investments, or an investment should contain a predominant portion of investments directed to a specified technology or industry. In this manner, some funds of a specific time horizon H.sub.t may be established which are tailored in accordance with investor-specified attributes.

A relationship may be established between one or more investment portfolios P.sub.m and each investment fund F.sub.n. The portfolio P.sub.m has an associated portfolio characteristic, or parameter, that may be predetermined either by the fund manager, the market, or by the individual investor. The portfolio characteristic typically is determined by or defines the type of assets constituting the portfolio. These assets may be characterized by such factors as the expected volatility of the investments, the expected responsiveness to market conditions, and expected return on the investments. Thus, an investment fund F.sub.n having a particular time horizon H.sub.t typically consists of several portfolios, each portfolio having a different characteristic. Alternatively, a relationship is established between one or more investment assets A.sub.k, without the use of a portfolio. In that embodiment, each investment asset A.sub.k is handled in a manner similar to a portfolio P.sub.m.

The portfolios P.sub.m of FIG. 1 may each consist of one or more investment assets A.sub.k, also called an asset mix, each representing a major asset class. These assets may be stocks, bonds, paper, currencies, gold, silver, precious metals and minerals, oil, gas and the like. Assets may be combined in a manner that achieves the predetermined portfolio characteristic.

In one embodiment, and as shown in FIG. 2, each fund F.sub.n may include a portion or value known as present value of future cash flow P.sub.vf. This portion represents a "phantom asset", or the total value of all future cash flows that are expected to be received for the fund F.sub.n. In that illustrated embodiment, the fund F.sub.n has a current balance C.sub.B representative of the current value of the portfolios (P.sub.1 -P.sub.5), including any cash available for investing. Available cash is removed from the P.sub.vf as it becomes available for investing in the portfolios of the particular fund F.sub.n.

One aspect of the present invention is maintaining a total fund balance, including P.sub.vf and C.sub.B, of a particular fund, while changing the investment mix of that fund over the time horizon for that fund. One way in which the present system may administer the investment funds, each fund having a fixed time horizon, is to adjust the investment mix in accordance with some identified criteria.

In one embodiment of the invention, and as illustrated in FIG. 2, the investment mix is adjusted by adjusting the percentage of available cash distributed among the portfolios P.sub.m in each fund F.sub.n. As shown, cash initially is invested in the portfolios P.sub.1 -P.sub.5 in accordance with a

certain percentage. In the illustrated embodiment, P.sub.1 has a relatively high-risk characteristic, whereas P.sub.5 has a relatively low-risk characteristic. While the fund is young, i.e., the P.sub.vf (60%) is greater than the cash balance C.sub.B (40%), the individual portfolios have a greater relative percentage of cash invested in the higher-risk portfolios P.sub.1 and P.sub.2. Over the maturity of the fund, the cash is distributed among the portfolios such that near the time horizon for the fund, the portfolios have a greater relative percentage of value invested in the lower-risk portfolios P.sub.4 and P.sub.5.

In a preferred embodiment, the change in distribution of available cash varies as a function of a risk tolerance R.sub.I which changes over the length to horizon L.sub.H, as previously described. Other processes that consider the time horizon of the fund may be used in conjunction with the present system.

In another embodiment, the portfolio mix or asset mix for a given fund is adjusted as a function of the length to horizon L.sub.H by exchanging individual ones of the portfolios P.sub.m or investment assets A.sub.k for portfolios or investment assets having different characteristics. Typically the exchange is between portfolios or assets having higher-risk characteristics for ones having lower-risk characteristics. As with the illustrated embodiment of FIG. 2, these changes may be made as some function of the time horizon L.sub.H and risk tolerance R.sub.I, depending on the particular system.

FIG. 3 shows a block diagram of software and hardware components for implementing one embodiment of the present invention 100. Processor 102 is a conventional engineering workstation or other computer processor such as an Intel 80.times.86 or Pentium central processing unit (CPU), Motorola 680.times.0 CPU, RISC CPU and the like. Processor 102 may also be coupled to other processors accessible over conventional communications channels or buses (not shown). Processor 102 is conventionally coupled to storage 104 which may be a magnetic disk storage, a CD storage unit, or other conventional computer data storage unit. Storage 104 may also be coupled to other storage units accessible over conventional communications channels or buses (not shown).

Processor 102 is also conventionally coupled to memory 108 which is a random access memory (RAM) unit or other conventional computer memory. Items in memory 108 may alternatively be stored in storage 104 and accessed by processor 102 when required. Memory 108 may comprise a fund database 110 for storage and retrieval of information related to various funds (e.g., H.sub.T, L.sub.H, C.sub.B, P.sub.vf, Risk Tolerance R.sub.I, and portfolios P.sub.1 -P.sub.m), investor database 112 for storage and retrieval of information related to various investors (e.g., risk preference; preferences or constraints on investments such as domestic investments only, environmentally conscious investments, technology areas of investment, or industry areas of investment), and a strategic investment module or program component 114 as discussed below. Strategic investment program comprises a plurality of program instructions executable on processor 102.

Input 101 comprises conventional input devices such as a keyboard, mouse, track-ball, or touchscreen. A conventional display unit 120 may also be conventionally coupled to processor 102.

The preferred embodiment of the present invention may be implemented on any platform, operating system, and user interface such as: IBM PC or Compatibles/Microsoft Windows; Sun/Sun OS-SunView; DEC VAX/VMS, and the like, and may be operated in a distributed data/distributed processor environment, if desired.

FIG. 4 is a flow chart of one embodiment of the initialization process of the present system. One or more investment funds F.sub.n are established 502 and stored 504 in storage 104. A time horizon H.sub.t and a length to horizon L.sub.H are associated 506 with each of the stored investment funds F.sub.n, and the data F.sub.n, H.sub.t, L.sub.H are stored 510 in storage 104. At the same time, or at a separate time, one or more investment assets A.sub.k may be established 512 and stored 514 in storage 104. These assets A.sub.k represent real tangible assets such as stocks, bonds, paper, currencies, gold, silver, precious metals and minerals, oil, gas and the like, which are bought on the market in order to establish the fund. Alternatively, step 512 may include establishing a portfolio

P.sub.m having one or more assets A.sub.k.

Each fund, F.sub.n, is associated with one or more assets, A.sub.k (or P.sub.m) 516, and stored 518 in storage 104. Cash or other contributions can then be directed 520 to the various investment assets A.sub.k (or portfolios P.sub.m) associated with each fund F.sub.n and distributed among the assets A.sub.k (or portfolios P.sub.m) as a function of the length to horizon L.sub.H associated with each fund F.sub.n, as described in detail above. The initial distribution 520 of cash among the investment assets A.sub.k or portfolios P.sub.m establishes the investment mix, of each fund F.sub.n. It is this investment mix that is adjusted 522 in accordance with some predetermined criteria, as described in further detail below, which criteria also is a function of the length to horizon L.sub.H associated with the fund F.sub.n.

In a preferred embodiment, the investment mix of a fund F.sub.n is adjusted 522 as a function of the risk R.sub.I, where R.sub.I varies with the length to horizon L.sub.H associated with the fund F.sub.n. By associating the time horizon H.sub.t (or L.sub.H) with a risk level or preference indicator, such as R.sub.I, it is possible to decrease the amount of risk in investments over the life of a particular fund. For example, a fund F.sub.1 that has an H.sub.t of 40 years may have an initial investment mix that has high expected risk because the investor has high risk tolerance R.sub.I. Conversely, a fund F.sub.2 may have an associated H.sub.t of 10, in which the R.sub.I is quite low, leading to conservative investment mix. The risk of the fund is conventionally determined by statistical variance analysis or fluctuation of the fund in relation to the market.

In practicing a preferred embodiment of the present invention, and as shown in FIGS. 5A-5D, once a fund F.sub.n is established, the risk tolerance R.sub.I shifts over time without transferring any cash out of the fund. The illustrated fund F.sub.n has a time horizon H.sub.t of 40 years. In the first year, FIG. 5A, the R.sub.I is high, thus the investment mix will reflect the aggressive investment approach. The investments mix is thus proportionately in higher-risk funds. Over time, as L.sub.H diminishes, the R.sub.I shifts and the investment strategy becomes more risk averse, as illustrated in FIG. 5B. Continuing, the R.sub.I decreases until a more conservative, or less aggressive, investment mix is achieved, as illustrated in FIGS. 5C and 5D. The investments mix is thus proportionately in lower-risk funds.

FIG. 6 is a general flow chart of a preferred embodiment of practicing the adjusting step 522 of the present invention. In the illustrated embodiment, information is obtained 700 from an individual investor regarding investor portfolio information and market data 800 is obtained regarding the market. The step of obtaining 700 investor portfolio information includes establishing a current portfolio 702, which becomes the repository for assets and cash. Investor assumption data, such as investment preferences (domestic limitation, technology focus, industry focus, environment conscious, and the like) may also be obtained 704.

The step of obtaining 700 investor portfolio information also includes obtaining 706 risk tolerance data. Such data includes both the investor policy risk tolerance, which typically is a measure of an aggressive/conservative investment policy on behalf of the investor, and a market response coefficient, which typically is a measure of whether the investor is generally contrary, market neutral, or insurance oriented. A risk tolerance R.sub.I is determined 708, which is used in determining 812 portfolio risk, described in further detail below.

Finally, the step of obtaining 700 investor portfolio information may include obtaining 710 contribution data. That data may include such factors as the investor's current salary, expected growth rate of that salary, the investor's contribution rate, any matching funds contributed by a third source, the investment horizon H.sub.t, plus any outstanding or current fund balance. Other factors may be included or omitted as appropriate for an individual investor or system. Each of these factors may be used, in combination or individually, to calculate 712 the anticipated cash flow stream for the investor.

The step of obtaining 800 market data preferably includes the steps of obtaining 802 interest rate data and obtaining 804 other market data. The step of obtaining 802 interest rate data may include

calculating 806 some discount function that then may be used, in combination with the anticipated cash flow stream produced in step 712, to determine 808 the present value of future cash flow P.sub.vf, in the form of future cash flows.

The step of obtaining 804 other market data may include determining market risk premiums, expected returns, and transaction costs associated with buying/selling portfolios and/or individual assets, which may include obtaining alphas for each portfolio and/or each asset. An alpha is defined as the difference between the investor's expected return and the observed market consensus expected return for any particular portfolio or asset. Other values may be obtained 804 as appropriate for the particular system and investors. The market data may be used to forecast 810 market risks and returns, and to determine 812 the portfolio risk.

The fund then is optimized 900 in accordance with any commercially available optimizer program or system, such as AAT, available from Scientific Press (So. San Francisco, Calif.). Inputs to an optimizer program 900 typically include a lower bound, which may be set to be that portion of the portfolio which represents the present value of all future contributions to the portfolio by the investor, or set in accordance with the investors' desired minimal exposure to an asset class or portfolio. Optimizers 900 typically also include an upper bound, which represents a maximum exposure that any investor wishes to have to one or more asset classes or portfolios. Other inputs of available optimizers typically include: asset and/or portfolio characteristics, such as expected returns risks, and correlations; transactions costs of all types; current holdings; and investor risk tolerance. Other parameters may be considered or omitted, depending upon the particular optimizer used in conjunction with the present invention.

The present system may further include creating 902 an investor trade list. The trade list may be in the form of output from the optimizer 900, and lists the assets that need to be exchanged to obtain an optimal mix of investments. The investor portfolio may then be rebalanced 904 by trading, selling and/or purchasing assets in the open market, then maintained 906 for a specified period, such as for one month. Typically, trade lists are a necessary part of the process even when trading activity is motivated by contributions or withdrawals of cash.

FIG. 7 illustrates a high-level flowchart of one embodiment of the investment management module 114 shown in FIG. 3. Current asset classes 701 for a fund are input to the system for processing. In a preferred embodiment, 17 asset classes 701, or style indices (which represent assets), are input as shown in Table I. Asset classes represent tangible commercial assets or goods such as bonds, stocks, money markets, currencies, cash, gold, silver, oil, gas, or other precious metals and minerals. As the investment mix changes according to the present investment, these goods are physically purchased or disposed of. Asset classes and style indices are used interchangeably herein.

TABLE I

Domestic Equity
Large Growth (Equity Growth)
Intermediate Value
Intermediate Growth
Intermediate Utilities
Small Value
Small Growth
Micro
International Equity
MSCI EAFE x-Japan*
MSCI Japan
Domestic Fixed Income
Intermediate Government
Long Government
Intermediate Corporate
Long Corporate
Mortgage Backed Securities
International Fixed Income

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Salomon non-US WGBI*
Cash
Treasury Bills (MOEBT*)
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*MSCI EAFE = Morgan Stanley Capital International Europe, Australia and
Far East Index
MOEBT = Well Fargo money market fund
WGBI = World Government Bond Index
```

The domestic equity style indices are a mutually exclusive and exhaustive set of style indices which combine (using capitalization weights) to form the US Equity Market Index. The domestic fixed income style indices are a mutually exclusive and exhaustive set of style indices which combine (using capitalization weights) to form the Lehman Brothers Aggregate Bond Index (minus asset backed securities). The international equity style indices combine to form the MSCI EAFE Index. The international fixed income index forms the Salomon Brothers non-U.S. WGBI. The final asset class is cash. Other similar asset classes or style indices 701 can also be used. It is assumed that expected returns, risks, and correlations of asset classes 701 change infrequently, and the markets are in equilibrium.

A strategic investment strategy is determined 703, which produces strategic asset class weights 705 for each asset class 701. In one embodiment, the strategic component 703 represents 75% of the overall investment strategy of a fund (e.g., 75% of incoming cash or other investment is invested strategically), however this percentage can be modified as desired. Determination of the strategic asset class weights 705 is discussed below with reference to FIG. 8.

Strategic asset class weights 705 are used to modify 711 the mix of assets in a fund 713 to produce a modified asset mix for a fund 715. Based on this modified asset mix 715, assets in the fund are bought or disposed of on the open market to conform to the strategic asset mix 705. New investments coming into the fund (e.g., cash, contributions, electronic funds) are also allocated according to the strategic asset mix 715. The strategic investment component (701, 703, 705) can be repeated in order to process a plurality of funds.

Strategic asset class weights 705 are used to determination a tactical investment strategy 707. In one embodiment, the tactical component 707 represents 25% of the overall investment strategy of a fund (e.g., 25% of incoming cash or other investment is invested tactically), however this percentage can be modified as desired. The tactical investment component can be repeated in order to process a plurality of funds. The tactical component 707 is optional. Determination of tactical investments 709 is discussed below with reference to FIG. 9.

FIG. 8 is a flowchart illustrating the determination of the strategic asset class mix weights 703. A correlation matrix is conventionally determined 803 for the asset classes in a fund 701 (e.g., assets listed in Table I). In one embodiment, the monthly returns (equal weighted) for the previous ten calendar years (120 months) of each asset class 701 are used to determine a conventional correlation matrix 803. In a preferred embodiment, correlation matrix 803 is updated on an annual basis.

The risk of each asset class 701 is conventionally determined 805 using the standard deviation of the monthly returns (equal weighted) for the previous ten calendar years (120 months) of the asset class 701. In a preferred embodiment, the risk estimates 805 are updated on an quarterly basis.

Implied expected returns for each asset class are determined 807 conventionally using zero beta CAPM (Capital Asset Pricing Model). Given the expected risk 805 of each asset class, the correlation matrix 803, an efficient portfolio 809, and the expected returns on two asset classes (E.sub.1, E.sub.2) 811, 813, the implied expected returns, E.sub.i, for each asset class 701, i, can be calculated as follows:

E.sub.i =z+.beta..sub.i *p

where

E.sub.i =the expected return of asset class i

.beta..sub.i =the beta of asset class i

p=(E.sub.1 -E.sub.2)/(.beta..sub.1 -.beta..sub.2) and

z=E.sub.1 -.beta..sub.1 *p

.beta..sub.i for all i classes 701 is known from the covariances and variances of each asset class in the correlation matrix 803. In one embodiment, the expected return of one known asset class, E1, can be estimated using a conventional dividend discount model (DDM is the consensus expected dividend to calculate the expected return of the security) and attributing it to the Large Value asset class. The second known expected return, E2, can then be estimated using the average historical spread between stocks and bonds to determine an estimate of the equilibrium spread between stocks and the Long Governments asset class. For example, if the historical spread is 3.00%, the expected return for the Long Government asset class is equal to the expected return for the Large Value asset class minus 3.00%.

A proxy for an efficient portfolio 809 is the average strategic asset mix of several (e.g., twelve) large defined benefit plans (e.g., with combined total assets of $250 billion) sampled annually. It is assumed that these plans are a good proxy for a sophisticated investor. At the broad asset class category level (see Table I categories of asset classes) this `sophisticated investor' efficient portfolio mix as of Dec. 31, 1993 was 51.1% domestic equity, 19.3% international equity, 27.3% domestic bonds and 2.3% international bonds. Within the domestic asset classes (equity and bonds) these broad asset classes are sub-divided into the asset classes 701 based on relative market capitalization. For international equity, the relative weights are 75% EAFE ex-Japan and 25% Japan to reflect the underweighting (relative to market-capitalization) of Japan in US institutional equity portfolios. It is estimated that 20% of international equity exposure is indexed and 80% is actively managed. Using a market capitalization weighting of 45% for the indexed component and a weighting of 20% for the average active equity manager, the Japan weighting is 25% for the average US institutional investor.

Given the inputs of an asset class correlation matrix 803, asset class risks 805, and asset class expected returns 807, an efficient frontier 815 is conventionally determined using a mean/variance optimizer. The efficient frontier 816 represents the optimal mix of a given set of asset classes for any given desired risk, R, of investment. Each point on efficient frontier 815 represents a portfolio or mix of assets whose expected return is the highest for that given level of risk. A commercially available product, such as Ibbottson Associates' EnCORR/Optimizer can be used to calculate efficient frontier 815.

In a preferred embodiment, constraints 817 on selected asset classes can be imposed during the calculation of efficient frontier 815 such that an optimal class weight mix does not recommend more than a specified percent of the selected asset class. Asset class constraints 817 are generally only implemented in the strategic investment strategy component. For example, in a preferred embodiment, an international constraint is imposed so that no more that 26% (0.26 weight) is allocated to international assets, and, no more than 15% (0.15 weight) is allocated to cash:

international constraint:

.sup.W Japan+.sup.W EAFE-ex Japan+.sup.W intl bonds.ltoreq.0.26 (26%)

cash constraint:

.sup.W cash.ltoreq.0.15 (15%)

Before, after or during the calculation of the efficient frontier 815, the optimal risk level, R.sub.I, 820 is determined 819. As discussed above with reference to FIGS. 5A-5D, the risk level, R.sub.I, generally decreases as a function of L.sub.H the length of time remaining between the present time and the time horizon H.sub.t 821. If returns are independent across time, then risk (standard deviation annualized) will change with the square root of time.

R.sub.I =.sqroot.L.sub.H *F

where L.sub.H =time in years to the maturity of the fund (821), and F=a constant risk factor (823). The constant risk factor, F, 823 is determined such that the recommended mix is 100% equity allocated across several domestic and international equity asset classes at the beginning of a fund (e.g., L.sub.H =50); and the allocation is 20% equities at the mature state (e.g., L.sub.H =0 or L.sub.H =4). In one embodiment, F=2.15. If asset class risk levels or final desired allocations change then F should be adjusted.

In one embodiment L.sub.H 821 is equal to 50 at the inception of a fund, and for the mature state it is assumed to be 4 years. This is to ensure that the allocation at the mature state has approximately 20% equities. Thus,

```
R.sub.I = .check mark.L.sub.H *F
                     for L.sub.H = 50 to 5
R.sub.I = .check mark.(5-((5-L.sub.H)/5))*F
                     for L.sub.H < 5
```

The strategic asset class mix or weights 705 for a given fund is determined 825 by finding the mix on the efficient frontier 815 corresponding to the optimal risk level, R.sub.I, 819 for the fund. In one embodiment, strategic asset class weights 705 are modified such that the strategic component of the process (703, 705) accounts for a given percent of the overall investment strategy for a fund, e.g., 75 percent in a preferred embodiment. This step is accomplished conventionally by multiplying the strategic asset class weights by a factor less than 1 (e.g., 0.75).

FIG. 9 is a flowchart illustrating the determination of the tactical investment strategy 707. The tactical component 707 is designed to improve the fund performance by taking advantage of relative mis-pricing across asset classes. Markets are not assumed to be in equilibrium, and expected returns are updated daily. In one embodiment, a certain proportion of investments directed to a fund represent tactical investments, e.g, 25% in a preferred embodiment. The tactical component 707 is used to determine how to invest the 25%.

In one embodiment, the tactical investment component 707 of a fund is divided between two commercially available allocation strategies--the Wells Fargo U.S. Tactical Asset Allocation (100/0/0, stocks/bonds/cash, in normal market conditions) and the Wells Fargo U.S. Treasury Asset Allocation (bonds, notes, cash). The strategic asset class weights 705 are used to determine 901 how much should be invested according to the Tactical Asset Allocation 903, and how much should be invested according to the Treasury Asset Allocation 905. In this embodiment, the percent of strategic asset class weights 705 allocated to equities (e.g., the weights allocated to asset style indices or asset classes under the Domestic Equity and International Equity categories, plus Intermediate Corporate, Long Corporate, Mortgage Backed Securities, and Cash--see Table I above) is invested according to the Tactical Asset Allocation 903, while the percent of strategic asset class weights allocated to non-equities (e.g., (100- % equity) or the percent in International Fixed Income, Intermediate Government, Long Government, and Salomon non-US WGBI--see Table I above) is invested according to the Treasury Asset Allocation 905.

The Tactical Asset Allocation comprises the following indexes for given asset class categories:

Asset Class	Index
Stocks	S&P 500 Index
Bonds	Lehman Brothers 20+ Treasury Bond Index
Cash	MOEBT

The U.S. Treasury Asset Allocation strategy comprises the following indexes for given asset class categories:

Asset Class	Index
Bonds	Lehman Brothers 20+ Treasury Bond Index
Notes	Lehman Brothers 5-7 year Treasury Index
Cash	Treasury Bills

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United States Patent **5,884,287**
Edesess **Mar. 16, 1999**

System and method for generating and displaying risk and return in an investment portfolio

Abstract

The present invention is a computer-implemented system and method to create an optimal investment plan given wealth goals stated in probabilistic form, and to display the resulting probability distributions of wealth accumulations at future times where the method provides inputs for entering and storing in a computer target and fallback scenarios and required probabilities, computes rate of return values responsive to the user input, generates an efficient portfolio array, computes probabilities for the efficient portfolio array related to the rate of return values, iteratively compares the array probabilities so that the target and fallback scenario probabilities are satisfied and an optimum efficient portfolio is selected and then provides a graphical representation of the selected efficient portfolio.

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References Cited | [Referenced By]

U.S. Patent Documents

4,839,804	Jun., 1989	Roberts	364/408
4,953,085	Aug., 1990	Atkins	364/408
5,132,899	Jul., 1992	Fox	364/408
5,148,365	Sept., 1992	Dembo	703/36
5,222,019	Jun., 1993	Yoshino	364/408
5,237,500	Aug., 1993	Perg	364/408
5,471,575	Nov., 1995	Giansante	395/144
5,563,783	Oct., 1996	Stolfo	364/408

Other References

William N. Goetzman, "An Introduction to Investment Theory", Yale School of Management, Apr. 9, 1997, Class Notes.

Keyes, Jessica. "Expert Allocator: Tools for portfolio optimization." Pension Management, pp.

44-46, May 1996.

Putnam, Bluford. "Asset allocation--one step at a time." Global Investor, No. 100, pp. 67-69, Mar. 1997.

"Ibboston Associates offers asset allocation program." Bank Mutual Fund Report, vol. 4 No. 11, Jun. 1994.

Edessess, Michael et al. "Scenario forecasting: Necessity, not choice." Journal of Portfolio Management, vol. 6 No. 3, pp. 10-15, Sprg 1980.

"Keeping up with technology: the 1991 software update" Trusts & Estates, pp. 34-67, Jun. 1991.

Paroush, Jacob. "Risk and wealth effects on efficient portfolio." Metroeconomics, vol. 26, No. 1-3, pp. 86-96, 1974.

Voros, J. "Portfolio analysis--an analytic derivation of the efficient portfolio frontier." European Journal of Operations Research, vol. 23, No. 3, pp. 294-300, Mar. 1986.

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14 Claims, 7 Drawing Figures

United States Patent *5,799,287*
Dembo **August 25, 1998**

Method and apparatus for optimal portfolio replication

Abstract

A method and apparatus for determining an optimal replicating portfolio for a given target portfolio involves an initial step wherein a user defines a target portfolio to be replicated, a set of available market instruments from which the replicating portfolio may be created, a set of future scenarios, a horizon date, and a minimum profit to be attained. A representation of the trade-off between risk and expected profit for some arbitrary replicating portfolio is then determined and used to calculate a maximum risk-adjusted profit. The maximum risk-adjusted profit reflects that level of return that may be achieved with an optimum degree of risk; that is, it reflects that point in the risk/reward trade-off where a marginal cost of risk is equivalent to a marginal benefit attainable by assuming that risk. The method then uses the predefined set of available market instruments to identify a set of transactions that will create a replicating portfolio that will achieve the maximum risk-adjusted profit. The method and apparatus also derives the information required to compute a risk premium for pricing of portfolios in incomplete markets, and performs the computation.

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Intern'l Class:	G06F 157/00
Field of Search:	705/36,37,35 395/925

References Cited [Referenced By]

U.S. Patent Documents

4346442	Aug., 1982	Musmanno	364/408.
4642768	Feb., 1987	Roberts	364/408.
4674044	Jun., 1987	Kalmus et al.	364/408.
4694397	Sep., 1987	Grant et al.	364/408.
4722055	Jan., 1988	Roberts	364/408.
4744026	May., 1988	Vanderbei	364/402.
4744027	May., 1988	Bayer et al.	364/402.
4744028	May., 1988	Karmarkar	364/402.

4752877	Jun., 1988	Roberts et al.	364/408.
4797839	Jan., 1989	Powell	364/554.
4953085	Aug., 1990	Atkins	364/408.
5101353	Mar., 1992	Lupien et al.	705/37.
5148365	Sep., 1992	Dembo	364/402.

Other References

Goddard, Claire, Measuring Treasury Performance, Mortgate Finance Gazette, May 6, 1992, pp. 1-2.
World: Forex-A World of Options, Euromoney Supplements, Nov. 17, 1989, p. 3.
Stein, Jon, Where Corporate Treasurers Can Look for Help, Futures, Oct. 1989.
"Derosa ARBS Japanese Warrants Against Nikkei Options", Derivatives Week, May 4, 1992, vol. I, No. 5, p. 2.
Wiest, "Portfolio Gains Favor in FX Mangement", Reuter's BC Cycle, Apr. 6, 1992.
Elgin, "Portfolio Hedging Emerges in New Forms to Shield Investments: Investments and Benefits", Corporate Cashflow Magazine, vol. 11, p. 22.
Hansell, "Is the World Ready for Synthetic Equity", Institutional Investor, Aug. 1990, p. 54.
Voorkees, "Can Portfolio Insurance Make a Comeback", Institutional Investor, Jan. 1988, p. 57.
Ring, "Wells Fargo Gets Most New Business in Dynamic Wedging", Pensions & Investment Age, Feb. 9, 1987, p. 31.
Dembo et al., "Tracking Models and the Optimal Regret Distribution in Asset Allocation", Applied Stochastic Models and Data Analysis, 1990, vol. 8, pp. 151-157.
Dembo, "Scenario Optimization", Annals of Operation Research, 30 (1991) pp. 63-80.
Torres, "`Synthetic` Stock: Future Stand-In for the Real Thing", The Wall Street Journal, Oct. 19, 1990.
Torres, "Mathematician Race to Develop New Kinds of Trading Instruments", The Wall Street Journal, Oct. 18, 1991.
Stoffman, "The Hedging Hotshot of Bay street", Dec. 1990, pp. 56-59.
Dembo et al., "French Dressing", Equity Derivatives.
Dembo et al., "Protective Basket," Options, vol. 3, No. 2, Feb. 1990, pp. 25-28.
Dembo, "The Art of the Qptimum", Risk, vol. 3, No. 4, Apr. 1990, pp. 17-21.
Dembo et al., "Share the Load", Risk, vol. 4, No. 4, Apr. 1991, pp. 44-47.
Davidson, "Drawing on Data", Technology, pp. 49-53.
"The Cafe Behind the Hedge", Open Finance, Summer 1992.

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Parent Case Text

This is a continuation of application Ser. No. 08/248,042, filed May 24, 1994, now abandoned.

Claims

1. A computer-based method for constructing an optimal replicating portfolio for a given target portfolio of market instruments, the method comprising the steps of:

(a) generating an electronic representation of the collection of market instruments;

(b) generating an electronic representation of a set of available market instruments from which the

replicating portfolio may be constructed;

(c) defining a set of future scenarios, wherein each member of the set of future scenarios associates a future value with a market parameter;

(d) defining a horizon date and a minimum required profit to be obtained on the horizon date from a replicating portfolio for the given target portfolio;

(e) calculating a trade-off between risk and expected profit for an arbitrary replicating portfolio;

(f) calculating a maximum risk-adjusted profit using the set of future scenarios and the trade-off between risk and expected profit, wherein the maximum risk-adjusted profit corresponds to a marginal cost of risk that is equivalent to a marginal benefit to be obtained from assuming that risk;

(g) generating an electronic representation of a replicating portfolio for the given target portfolio that will achieve the maximum risk-adjusted profit, wherein the replicating portfolio comprises market instruments selected from the set of available market instruments; and

(h) identifying a set of transactions required to construct the replicating portfolio.

2. The method according to claim 1, further comprising the step of computing a risk premium caused by an inability to construct a replicating portfolio that is a perfect replication of the target portfolio.

3. The method according to claim 1, further comprising the step of computing a price for the replicating portfolio by discounting a future price of the replicating portfolio according to a current price of a known market instrument.

4. The method according to claim 3, wherein said step of computing a price for the replicating portfolio further comprises calculating a state price vector and applying the state price vector to the future price of the replicating portfolio.

5. The method according to claim 1, further comprising the step of executing the set of transactions to construct the replicating portfolio.

6. A computer-based apparatus for constructing an optimal replicating portfolio for a given target portfolio of market instruments, the apparatus comprising:

(a) an input module programmed to accept information including an electronic representation of the target portfolio, a set of future scenarios, an electronic representation of a set of available market instruments, and a minimum profit to be achieved;

(b) an optimization module programmed to calculate an optimal replicating portfolio associated with the target portfolio, the set of future scenarios, and the minimum profit to be achieved, wherein the optimal replicating portfolio comprises an electronic representation of a set of market instruments for which a marginal cost of risk is equivalent to a marginal profit to be obtained from assuming that risk;

(c) a portfolio replication module programmed to identify a set of transactions that will construct the optimal replicating portfolio from a subset of the set of available market instruments; and

(d) an output module programmed to provide a user of said apparatus with information relating to the optimal replicating portfolio.

7. The apparatus of claim 6, further comprising a price calculator programmed to compute a current price for a financial instrument consistent with a current price of a known market portfolio.

8. The apparatus of claim 7, wherein said price calculator computes a state price vector for

discounting the future price of the financial instrument.

9. The apparatus of claim 6, wherein said optimization module is further programmed to calculate a trade-off between the minimum profit to be achieved and a degree of risk required to achieve that minimum profit.

10. The apparatus of claim 6, wherein said input module comprises a real-time data feed.

11. A computer-based apparatus for constructing an optimal replicating portfolio for a given target portfolio of market instruments, the apparatus comprising:

a memory;

an input device for receiving input, said input device including

means for generating an electronic representation of a target portfolio,

means for generating an electronic representation of a set of available market instruments from which the replicating portfolio may be constructed,

means for defining a set of future scenarios, wherein each member of the set associates a future value with a market parameter, and

means for defining a horizon date and a minimum required profit attainable from the replicating portfolio at the horizon date; and

a processor coupled to the input device and the memory, said processor including

means for determining a trade-off between risk and expected profit for an arbitrary replicating portfolio,

means for determining a maximum risk-adjusted profit using the set of future scenarios and the trade-off between risk and expected profit, wherein the maximum risk-adjusted profit corresponds to a marginal cost of risk that is equivalent to a marginal benefit to be obtained from assuming that risk,

means for generating an electronic representation of a replicating portfolio for the target portfolio that will achieve the maximum risk-adjusted profit, wherein the replicating portfolio comprises market instruments from the set of available market instruments, and

means for identifying a set of transactions required to construct the replicating portfolio.

12. A method for determining whether a given market instrument is fairly priced, the method comprising the steps of:

(a) generating an electronic representation of the given market instrument;

(b) generating an optimal replicating portfolio for the given market instrument, wherein the optimal replicating portfolio represents a maximum risk-adjusted profit under a predetermined set of future scenarios, the maximum risk-adjusted profit corresponding to a marginal cost of risk that is equivalent to a marginal benefit to be obtained by assuming that risk, the optimal replicating portfolio comprising an electronic representation of one or more predetermined market instruments;

(c) determining a value corresponding to the maximum risk-adjusted profit associated with the replicating portfolio; and

(d) indicating whether the given market instrument is fairly priced based on the value of the maximum risk-adjusted profit, wherein the given market instrument is deemed to be overpriced when

the value is positive, fairly priced when the value is zero, and underpriced when the value is negative.

Description

FIELD OF THE INVENTION

The present invention is directed to a computer-based method and apparatus for replication of a portfolio; and in particular, to a method and apparatus for generating a replicating portfolio with an optimal balance of expected profit and risk.

BACKGROUND OF THE INVENTION

A portfolio manager controls a portfolio (or "book") of equities or other securities that are usually traded on an exchange, such as the New York Stock Exchange. The portfolio manager must continuously adjust the book by making trades aimed at increasing reward (that is, profit) while reducing the risk of loss. In some cases, the portfolio manager may decide to undertake a particular risk where the risk is slight compared to the potential reward. As the portfolio manager makes trades, the risk and potential reward values of the portfolio adjust according to the characteristics of the changing portfolio assets.

One way in which portfolio managers in the investment field attempt to increase portfolio performance and profit while controlling risk or exposure to loss is through portfolio insurance. Various approaches to portfolio insurance are known, such as hedging, dynamic hedging, and option or replication techniques. The principle focus of such techniques is to provide downside protection for any exposure inherent in the portfolio; in other words, portfolio insurance guards against the risk of a dramatic downturn in the value of the portfolio.

A significant disadvantage of present hedging methodologies is their reliance on standard assumptions that are often incorrect. These standard assumptions typically include (a) the price of an underlying asset may be represented by a continuous random variable; (b) there are no transaction costs; (c) the market is liquid, complete and arbitrage-free (that is, instruments are correctly priced); (d) volatility of price movements is fixed or may be represented as a known function of time; and (e) discount rates are fixed or may be represented as a known function of time.

Using assumptions such as these, Black and Scholes developed a dynamic trading strategy designed to perfectly replicate the payoff function of a European option. See F. Black and M. Scholes, The Pricing of Options and Corporate Liabilities, Journal of Political Economy, vol. 81, 637-654 (1973). The Black and Scholes replication strategy is the basis for the vast majority of hedging done today. Unfortunately, most markets are not continuous, nor is trading "frictionless" (that is, without transaction costs). Market gaps often occur, for example, as a result of price changes, volatility changes, and interest rate movements. For instance, a market gap would occur if the Federal Reserve were to announce an increase in its prime lending rate while a market affected by that rate, such as the New York Stock Exchange, was closed. In such a case, interest rates were at one value when the market closed, but would be at a second, higher value when the market reopened for the next session. As a result of such gaps, returns on investments are often far from the expected.

Insurance is generally desired over a time frame long enough to match the expected period of portfolio exposure. Ideally, the portfolio manager is able to acquire options which are sufficiently long-dated so as to provide the necessary insurance over the desired time period. Although long-dated options are available in the over-the-counter (OTC) market, the lack of liquidity and standardization for these options makes them more expensive than similar exchange-traded options. However, since exchange-traded options frequently have much shorter maturities than those desired for long-dated options, portfolio managers are nonetheless forced to acquire such options in the OTC market.

Current option or replication techniques enable a portfolio manager to create synthetic long-dated options that satisfy the requirements for portfolio insurance. A desired synthetic option may be

created from a combination of various existing cash market instruments, futures contracts, and exchange-traded options. The components of such synthetic long-dated options may be acquired with significantly less cost than equivalent long-dated options available over-the-counter, thus lowering the cost of portfolio insurance. Advantages of such option replication strategies over dynamic hedging strategies include significantly lower management overhead and greater predictability of transaction costs.

In addition to creating synthetic securities which may or may not have an equivalent in the market, a portfolio manager may attempt to construct a portfolio whose value tracks a given market index. For example, a portfolio may be constructed from a set of bonds whose values are intended to offset a set of future liabilities or to hedge against losses in a given portfolio. This technique is referred to as "portfolio replication."

As noted above, dynamic replication based on Black-Scholes theory is the primary means used to hedge options positions in practice today. Black-Scholes dynamic replication is often called "delta hedging." Such hedges, however, generally fail at the very time when hedging is needed most, since the assumptions upon which these hedges are based do not hold true in turbulent markets. Accordingly, a method of portfolio insurance is desired that considers all possible future states of the world. Such a method would thus be able to provide adequate protection in the face of turbulent market conditions.

In pragmatic terms, a portfolio manager controlling a given portfolio (i.e., a target portfolio) has the objective of constructing a replicating portfolio that behaves identically to the target portfolio under all possible future states of the world. Such a replicating portfolio is called a perfect replication. A perfect replication will produce a perfect hedge for the target portfolio; that is, a short position in the replicating portfolio coupled with a long position in the target portfolio will result in no net exposure. In real markets, however, perfect replication may not always be possible.

An approach to portfolio replication is disclosed in R. Dembo, Scenario Optimization, Annals of Operations Research, vol. 30, 63-80 (1991) and Dembo and King, Tracking Models and the Optimal Regret Distribution in Asset Allocation, Applied Stochastic Models and Data Analysis, vol. 8, 151-157 (1992), both of which are expressly incorporated herein by reference. This technique employs a regret function that measures the expected difference between the value of a given portfolio and an arbitrary target portfolio at maturity. In effect, the regret function measures what one can achieve with a decision today against what one could achieve with perfect foresight. With perfect foresight, all possible scenarios and their corresponding probabilities are known at the start of the period over which a portfolio is to be hedged. According to the regret function, a replicating portfolio with a zero regret value will perfectly match the target portfolio under all possible outcomes. The regret function is also useful for determining the value of residual or known risk in the replicating portfolio. In markets where zero regret is not possible, it is best to obtain a replicating portfolio that comes as close as possible to zero regret. This technique is further described in U.S. Pat. No. 5,148,365 to Dembo, entitled "Scenario Optimization," the disclosure of which is expressly incorporated herein by reference.

A drawback of portfolio replication, even when using the regret function, is that the technique ignores the cost of maintaining the hedge. Under Black-Scholes theory, for example, the cost of a hedge over the life of a deal is set to its fair market price; however, this relies on many assumptions, including the absence of transaction costs. In practice, the assumptions required by Black-Scholes have proven to be far from valid. Indeed, the cost of a given hedge may be significantly higher than what the theory predicts. Delta hedging may thus be extremely costly, even to the point of exhausting all profits from a particular trade. In extreme cases, the cost of a delta hedge may be high enough to erase the entire annual profits of a trading operation. Such situations have arisen often enough to warrant the search for an improved hedging technique.

Known techniques for portfolio replication neither control the cost of a hedge nor provide a mechanism for trading-off the cost of a hedge against the expected quality of protection the hedge offers. For example, a small increase in the cost of a hedge may greatly decrease the risk of loss.

Thus, an improved hedging technique must include a determination of the cost of the hedge or replication to enable calculation of an expected risk-adjusted profit from the portfolio or deal that is being hedged or replicated.

An ideal starting point for an improved hedging technique is a known method for optimally allocating available resources in a physical system using a mathematical model having at least one parameter of uncertain value, as disclosed by the above-referenced patent issued to Dembo. Optimization methods of this type determine a single solution to a desired equation which best fits a desired system behavior. While this result is beneficial, often what is desired is not a single best-fit solution, but rather a family of solutions where each solution in the family represents an expected reward for a given degree of risk.

Another desirable feature of an improved hedging technique would be the ability to detect a market instrument whose market value differs from its "true" value. Known systems and models are able to detect mispricing in a market to a limited extent. For example, it is known to determine whether a security is mispriced relative to market conditions. This condition is referred to as "absolute" mispricing. Typically, however, models used for portfolio replication are unable to determine whether a security in a given portfolio is mispriced relative to the portfolio itself, as opposed to market conditions. That is, a portfolio manager may wish to determine whether an instrument is fairly priced relative to the other instruments in a portfolio. This type of mispricing is essentially a subset of absolute mispricing, since any given portfolio is a subset of some market.

Finally, an improved hedging technique should enable a portfolio manager to determine an expected profit relative to the risk associated with attaining that profit. When trading securities, it is of course desirable to maximize profits with respect to each transaction. To this end, current hedging techniques value securities with respect to market value. Nonetheless, because each transaction involves a measurement of risk, it is more desirable to value securities with respect to risk. The improved hedging technique should thus seek to determine profit for a deal by maximizing a risk-adjusted profit.

A method and apparatus for optimal portfolio replication according to the present invention provides a portfolio manager with a hedging tool that incorporates these desired features.

SUMMARY OF THE INVENTION

The present invention provides an improved method and apparatus for portfolio replication which seeks to reach an optimal balance between expected profit and the risk involved in attaining that profit. In one embodiment, the present invention identifies a set of transactions required to achieve an optimal hedge by analyzing the portfolio replication according to a stochastic model which takes into account the trade-off between the cost of the hedge and the quality of protection it offers.

Unlike known delta hedging techniques, the present invention can produce a hedge that provides protection over a range of user-specified scenarios while explicitly accounting for the cost of the hedge. Hedge cost is computed as the expected profit or loss accounting for the buy and sell decisions required to maintain the hedge over its lifetime. Moreover, the present invention can describe the risk involved with the hedge as a function of the size of the profit to be taken out of the position, thus enabling a portfolio manager to selectively determine the optimal trade-off between assumed risk and expected profits.

A hedge is typically designed to provide protection over some predetermined time period. At the beginning of any such period, there is uncertainty about which one of an infinite number of possible future states will actually occur. Given a target return distribution (that is, the profit to be derived from the portfolio), the objective for a portfolio manager is to structure a replicating portfolio that tracks the target return (or any other attribute, such as volatility) under all possible scenarios. The present invention is accordingly directed to producing a hedge having a smallest possible tracking error, or residual risk. Although some degree of residual risk is inherent in the uncertainty of the future state of the world, the present invention does not require arbitrary bounds or improbable

assumptions to prescribe a solution. Accordingly, the replicating portfolio will conform to expectations much more closely than is possible using previously known techniques.

The present invention adopts a constructive approach that explicitly specifies the trades that a portfolio manager should undertake to replicate a target portfolio. As discussed above, hedging based on Black-Scholes theory performs poorly in markets exhibiting gaps in the behavior of market variables such as price, interest rates and volatility. A Black-Scholes replicating portfolio thus provides only limited protection that can be extremely costly in volatile markets. Unlike Black-Scholes, the present invention does not assume that markets behave in a continuous fashion. Instead, the portfolio replication approach of the present invention is based on the more realistic view that markets behave in discrete fashion.

In an advantageous variant of the present invention, a state price vector may be used to derive risk-neutral probabilities, and thus a risk-free discount rate, that may be used for risk-neutral valuation of market instruments. This valuation is advantageously free of investor preferences, thereby ensuring a more accurate result. Assuming a set of scenarios and replicating instruments with known correct prices, the state price vector may also be used to detect mispricing in a security or a portfolio in a given market over a given period of time, again permitting a portfolio manager to develop a replicating portfolio with optimum accuracy.

A representative embodiment of the method and apparatus according to the present invention is a computer-based system that generates a replicating portfolio in four steps: information gathering, preprocessing, optimizing, and pricing. In the information gathering step, a user identifies certain sets of instruments and relevant instrument attributes. For example, the user identifies a target instrument or portfolio of instruments that has an expected payoff at a specified rollover date corresponding to a desired profile, a set of instruments that may be used to create a hedge portfolio, a current portfolio (if one is held), and any new securities to be priced. In addition, the user specifies ranges of values for any uncertain parameters (for example, volatility, yields, beta) to be used in calculating the future value of the instruments specified. These ranges of values define the future states with respect to which the hedge, state price vector and risk/reward profile will be created. Finally, the user assigns a weight to each of the values in the ranges to indicate an estimate of the relative probability of a particular future state actually occurring.

In the preprocessing step, the system determines the probability of each future state as a function of the weights specified by the user in the previous step. In addition, the system determines the expected value at the rollover date of the current holdings (if any), the target portfolio and each instrument in the replicating set, for each of the future states. The maximum possible profit that can be extracted over the life of the trade is then determined. Using these values, the system then creates an instance of an optimization model designed to create a hedge portfolio that replicates the target portfolio with minimum tracking error under the specified future states and subject to a minimum profit requirement.

Following the preprocessing step, the system uses an optimization method to solve the optimization model, with the minimum required profit set to the previously-determined maximum possible profit. The result of this optimization is a replicating portfolio and a corresponding minimum regret, or tracking error. The replicating portfolio, minimum regret and minimum profit are then displayed to the user on an appropriate output device. The replicating portfolio describes a set of trades required to create a portfolio having a minimum tracking error for the required minimum profit. In other words, the replicating portfolio instructs the portfolio manager how to create a portfolio that ensures, to the maximum extent possible, that the required minimum profit will be achieved.

According to one embodiment of the present invention, the risk-adjusted profit associated with the replicating portfolio may then be maximized, and the state price vector and risk neutral probabilities determined and saved to a memory and/or displayed on an output device. It is possible that the optimization method may generate a replicating portfolio where the expected profit over the life of the trade does not exceed the minimum required profit. In such a case, the minimum required profit is reduced and the optimization procedure is repeated. Where the expected profit does exceed the

minimum required profit, it can be assumed that the minimum regret over all feasible values of minimum profit has been explored.

In the final step according to this embodiment, the state price vector may be used to establish a price for a new security that is consistent with a portfolio of instruments with known prices. This new security may be, for example, a synthetic long-dated option. The appropriate price of the new security is then output to an appropriate device.

The present invention, by using models which assume that a market behaves in discrete fashion, provides a portfolio manager with a set of specific buy/sell recommendations that maximize risk-adjusted profit. The portfolio manager may then execute these recommendations to create the replicating portfolio. In addition, the portfolio manager may obtain a risk/reward profile for a portfolio, a state price vector, a risk-neutral discount rate, and a computed value of risk. The present invention also enables a portfolio manager to determine if the instruments in a portfolio are fairly priced relative to the other instruments in the portfolio. As can be seen, a method and apparatus according to the present invention can generate a risk-reward trade-off for any portfolio, independent of the market.

The utility of the present invention is not limited to managing investment portfolios. Indeed, the techniques of the present invention can easily be adapted for application to any situation involving a risk/reward trade-off. For example, the present invention can be used by a distributor or retailer of goods to determine an optimal shipping strategy, determining the size and timing of shipments depending on anticipated demand. In such an application, the present invention can be used to select shipping schedules and warehouse locations by trading-off the risk of not meeting a demand against the expected profits.

As another example, the present invention can be applied in the production field to select an optimal number of production facilities by trading-off the risk of not meeting a schedule against the cost of production. In yet another application, the present invention can be used to control a reservoir schedule; that is, to determine an optimal release schedule for water in a reservoir by trading-off the risk of not meeting a demand for either water or electricity against the cost of alternative sources of generating electricity. Additionally, the present invention may be used to determine whether hydro-electric power or fuel power should be generated, depending upon an expected likelihood of precipitation. As will be readily apparent to persons skilled in the art, the present invention can be readily applied to find the optimal solution to virtually any real-world problem requiring a trade-off between anticipated risks and desired rewards.

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. 1 is a block diagram showing exemplary system hardware for a computer-based embodiment of the present invention.

FIG. 2 is a graphical illustration of the high-level programming objects of a computer-based embodiment of the present invention.

FIGS. 3-6 are graphs representing maximum risk-adjusted profit in various market situations.

FIG. 7 is a graphical summary of the risk/reward relationship.

FIG. 8 is a flow chart illustrating an embodiment of a method which may be used to generate buy/sell recommendations designed to maximize risk-adjusted profit for a given portfolio.

DETAILED DESCRIPTION

The present invention is directed to a method and associated apparatus whereby a user may determine an optimal course of action by selecting a suitable balance between a desired reward and the risk required to attain that reward. Referring now to the drawings, FIG. 1 illustrates in block diagram form

exemplary system hardware that may be used in a computer-based embodiment of the present invention. A central processing unit (CPU) 2 or other computer-based processor performs logical and analytical calculations. In this embodiment, the CPU 2 operates on a "UNIX" brand or other "POSIX"-compatible platform under "MOTIF/X WINDOWS," and is portable to most workstation environments, including "SUN/OS" and "DEC/ULTRIX" workstations. The CPU 2 is coupled to a memory device 8, such as a high-speed disk drive. An input device 4 is coupled to the CPU 2, enabling a user to enter instructions and other data. The input device 4 may include a keyboard, a mouse or a touch-sensitive display screen. An output device 6, such as a video display monitor, is provided to present textual and graphical information to a system user. The present invention ideally supports real-time data feeds 10 and is capable of executing application programs written in the "C++" programming language using object-oriented programming techniques.

While the present invention is applicable to virtually any real-world situation requiring a trade-off between reward and risk, for purposes of illustration the invention will be described in the context of an embodiment providing a computer-based system for use by an investment portfolio manager in creating a hedge against potential losses. In this context, a portfolio manager may use the system to create a replicating portfolio to serve as a hedge for a given target portfolio. The portfolio manager uses the input device 4 to enter information relating to the target portfolio, any current holdings available for use in generating a replicating portfolio, and the time period for which a hedge is desired. Additional information may be provided through the real-time data feed 10, and may consist of an interface to a real-time stock information service such as that provided by "REUTERS, LTD."

The input information is then supplied to an application program running on the CPU 2, and may also be stored in memory 8. The application program includes software modules capable of processing the input information to ultimately generate a set of buy/sell recommendations which the portfolio manager may execute to optimally insure the target portfolio against significant losses. These trading recommendations are based on decisions made by the portfolio manager relating to the optimal balance of anticipated reward and risk. Information may be presented to the portfolio manager on the output device 4 in the form of graphs, textual displays and printed reports.

Referring to FIG. 2, in terms of object-oriented programming an interactive system 16 providing the application functions of the representative embodiment includes a scenario optimization object 12 and a simulation object 14. The scenario optimization object 12 may receive input relating to current, replicating and target portfolios, future scenarios and user-specified weight values, prices of instruments, and values for attributes such as price sensitivity, time sensitivity and yield sensitivity. Using this input, the scenario optimization object 12 determines a set of reasonable scenarios and their probabilities. Alternatively, the scenario optimization object 12 may accept a user-defined set of scenarios and probabilities. The simulation object 14 permits a portfolio manager to perform "what-if" analyses for given prices, attribute values and scenario probabilities. The scenario optimization object 12 can be used separately or in conjunction with the simulation object 14.

In the embodiment of FIG. 2, the scenario optimization object 12 and the simulation object 14 are embedded in an interactive system 16. The interactive system 16 provides a front-end for users, and can be tailored for particular applications, such as determining optimal hedges using multi-scenario optimization of hedging positions. The interactive system 16 preferably includes a data management component and a graphical user interface. In other embodiments, either or both of the scenario optimization object 12 or simulation object 14 can be embedded in other interactive systems 16 for performing other functions.

An optimal portfolio replication system configured according to the present invention can provide a portfolio manager with the following useful information:

(a) a set of buy/sell recommendations that maximize risk-adjusted profit;

(b) a risk/reward profile;

(c) a state price vector;

(d) a risk-neutral discount rate; and

(e) a computed value of risk.

Each of these outputs are explained below; however, a complete understanding of the present invention is aided by an examination of the underlying theoretical principles. These principles are discussed with reference to an exemplary stochastic problem.

Assume that a market includes a finite number N of available instruments, each of which can only be traded in finite amounts without affecting their price. In other words, the market exhibits finite liquidity. Further assume that a hedge is desired for a single time period, and that there are a known, finite number of possible scenarios S that may occur over this period. Only one of these scenarios will have occurred at the end of the period; however, exactly which scenario will occur is unknown at the start of the period.

Each agent i in this market is characterized by individual preferences, which may be represented by a probability vector p.sup.i in a matrix R.sup.S whose components are the subjective probabilities of a future state occurring. Given any target portfolio in this exemplary market, the objective of a portfolio manager desiring a hedge is to find a replicating portfolio that behaves identically to the target portfolio for all possible future states; in other words, a perfect replication.

To create a satisfactory replicating portfolio, let the set (q.sub.1, q.sub.2, . . . , q.sub.N) represent the known price of these replicating instruments at the start of the hedge period. Additionally, let the set (d.sub.1j, d.sub.2j, . . . , d.sub.nj) represent the value of these replicating instruments at the end of the hedge period if scenario j were to occur (where j=1, 2, . . . , S). This set of scenarios, representing various combinations of values for different market factors, represents all of the uncertain events one would need to know in order to determine the state of the market at some point in the future. Techniques for determining such a set of scenarios are well known in the art, and thus are not explained in detail herein.

The notation d.sub.j denotes the N-dimensional column vector with entries (d.sub.1j, d.sub.2j, . . . , d.sub.nj) The N by S dimensional matrix D has as its columns the vectors d.sub.j (where j=1, 2, . . . , S). Similarly, the notation q denotes the N-dimensional column vector with entries (q.sub.1, q.sub.2, . . . , q.sub.N). In this model, the replicating portfolio is represented by a column vector x.

For purposes of this illustration, assume the target portfolio was bought at the beginning of the hedge period for a total cost c, and is subsequently sold at the end of the hedge period for a total price t.sub.s, where the vector t corresponds to the total sale price depending on which scenario actually occurs. As above, t denotes a column vector with entries (t.sub.1, t.sub.2, . . . , t.sub.S).

According to the relationship represented by the above matrix, a perfect replication results from a portfolio that satisfies the equation:

.SIGMA..sub.j d.sub.ij x.sub.j =t.sub.i ; (i=1, . . . S) (EQ 1);

or, in matrix form:

D.sup.T x=t (EQ 2),

where the superscript T denotes the transpose of the matrix D. Here, D.sup.T x represents the value of a replicating portfolio under each of the predefined future scenarios, while t represents the value of a target portfolio under all of these scenarios.

A so-called complete market is one in which there always exists a portfolio x that perfectly replicates an arbitrary target portfolio; that is, there always exists an x satisfying equation (2) for an arbitrary t. A complete market arises when the market is sufficiently rich so that there are always more

"independent" instruments than scenarios. Two given instruments are independent if their prices are not merely a simple multiple of one another. In other words, independent instruments exhibit different behavior (i.e., values) under different scenarios. Unfortunately, real markets are incomplete.

The above formulation does not determine if or when a perfect replicating portfolio exists in a given market. If a perfect replication does not exist, a portfolio manager will wish to create a "best," albeit imperfect, replication. Here, "best" refers to a smallest possible error in terms of some mathematical norm.

To this end, let E(.) denote an expectation operator. According to the teachings of the Dembo article and the Dembo and King article cited above, a regret function R is defined as:

R=E(.parallel.D.sup.T x-t.parallel.) (EQ 3).

Thus, the regret function measures the expected difference between the value of a given portfolio x and the target portfolio at some horizon time in the future. In other words, R represents a comparison between what one can achieve with a given decision today and what one could achieve with perfect foresight, since with perfect foresight all possible scenarios and their corresponding probability distributions would be known at the start of the hedge period. A replication portfolio with zero regret will perfectly match the target portfolio under all possible outcomes. Another interpretation of regret is the value of residual or known risk in the replicating portfolio.

A related function, termed downside regret DR, is defined as:

DR=E(.parallel.(D.sup.T x-t).sub.-- .parallel.) (EQ 4).

This function represents a form of regret in which only negative deviations from the target are considered. In practice, downside regret may be more useful to a portfolio manager since positive deviations from a target portfolio are generally considered desirable.

As noted above, in a complete market it is always possible to identify a replicating portfolio x such that regret R is zero for a given target portfolio and distribution of scenarios. Further, where zero regret is not achievable, it is desirable to obtain a replicating portfolio that is as close as possible to zero regret. Accordingly, a function termed minimum regret MR may be defined as:

MR=Minimize.sub.x. . . E(.parallel.D.sup.T x-t.parallel.) (EQ 5).

Complete markets are therefore characterized by MR=0 for all t, while incomplete markets have MR.gtoreq.0.

According to these relationships, it is possible to determine a minimum regret portfolio x*, being the portfolio with the smallest possible residual risk that can be obtained without perfect foresight. In terms of the regret function, x* is the optimal replicating portfolio that minimizes residual risk. Thus, a long position in the target portfolio and a short position in x* (or vice versa) is the best available hedge in the face of uncertainty.

The foregoing model, while theoretically useful, is nonetheless deficient. Like known techniques based on the Black-Scholes theory, the model has not taken into account the cost of the hedge itself. The trade-off between the cost of a hedge and the quality of protection it offers, however, is the very essence of a sound hedging methodology. A principle benefit of the present invention, representing a significant advance over the art, is that it enables a portfolio manager to accurately analyze this trade-off in making hedging decisions.

The cost of a hedge can be determined in a number of ways. For example, the cost of a hedge can be computed as the initial cost of purchasing the replicating portfolio, denoted q.sup.T x. This, however, does not account for the value of the portfolio at the horizon (that is, at the end of the hedge period). The measure used by the present invention is based on the expected profit or loss over the life of the

hedge, which may be computed as follows.

Consider, for example, an issuer (such as a bank that wants to issue an index-linked note) who sells a target instrument short at the start of a hedge period and covers the position with a multi-scenario hedge obtained by replication. The position can then be closed out at the end of the period. The accounting for this transaction would be:

Start of period:

income c from selling the target instrument

less cost q.sup.T x of purchasing the replicating portfolio

End of period:

expected payoff E(D.sup.T x) from selling the replicating portfolio

less expected cost E(t) of repurchasing the target to close-out the deal.

Thus, the present-day value of the expected profit from the deal is:

r.sup.-1 E(D.sup.T x-t)+(c-q.sup.T x) (EQ 6),

where 1+r is the interest rate over the hedge period.

According to the present invention, a parametric optimization function that describes the risk/reward trade-off can be described as follows:

MR(K)=Minimize.sub.x . . . r.sup.-1 E(.parallel.D.sup.T x-t.parallel.) (EQ 7)

subject to: r.sup.-1 E(D.sup.T x-t)+(c-q.sup.T x).gtoreq.K (EQ 8).

Inequality (8) states that the deal should be expected to make at least K dollars. The parameter K, the expected profit, may be positive or negative. Since MR(K) is an implicit function which is monotonic non-increasing in K, the higher the profit to be taken from the deal, the higher the residual risk (minimum regret). In short, MR(K) is the minimum cost of risk associated with a deal that will yield a desired profit K. As the risk changes, so too does the expected profit.

Unlike known methods of portfolio replication, the present invention enables a portfolio manager to choose an optimal expected profit K based on the level of risk the portfolio manager deems acceptable. This optimal value of K may be determined as follows. To make an expected profit of K, an issuer must expend a cost of risk equal to MR(K). Thus, the risk-adjusted profit for the deal is:

K-MR(K) (EQ 9).

To maximize this risk-adjusted profit, the system according to the present invention solves the equation:

Maximize.sub.K . . . K-MR(K) (EQ 10).

The solution to this problem may be represented as K*, which occurs at .lambda.=1, where .lambda. represents a marginal cost of risk that is exactly equal to a marginal benefit from assuming that risk. This condition is represented graphically in FIGS. 3-6.

FIG. 3 shows a graph that may be used to determine a maximum risk-adjusted profit for a given market. The horizontal axis 20 represents expected profit, while the vertical axis 22 represents residual risk (measured as minimum regret). This particular graph illustrates a complete market with

no arbitrage; that is, perfect replication is possible. The line 24 of the function K represents expected profit (in dollars). As can be seen, it is necessary to assume some degree of risk to attain an expected profit greater than zero. Line 28 represents the cost of the risk MR(K) for this market situation, which is calculated according to equation (5) above. As shown, the cost of the risk MR(K) is greater than zero for all expected profits K greater than zero. A portfolio manager will wish to determine the maximum risk-adjusted profit that can be made in this market situation. The maximum risk-adjusted profit will occur at the point where the difference between the expected profit K and the cost of the risk MR(K) is greatest. Graphically, this occurs where a perpendicular 32 drawn from line K to line MR(K) is longest. Here, point 34 represents the maximum risk-adjusted profit.

The portfolio manager will also be interested in the cost of a replicating portfolio that will achieve this maximum risk-adjusted profit. Here, the tangent to the curve MR(K) at K* (30) represents the shadow price of the expected profit constraint (8) and has a slope of 1. Since FIG. 3 shows a complete market with no arbitrage, the cost c of a perfect replicating portfolio x* (that is, the minimum regret portfolio) may be represented by c=q.sup.T x*.

Those skilled in the art will readily recognize the practical value of the present invention. A portfolio manager can use such information to determine the desirability of assuming additional risk beyond that associated with the maximum risk-adjusted profit. In the market of FIG. 3, for example, the portfolio manager will see that risk rises dramatically in relation to any additional profit that may be attained.

The graph of FIG. 4 represents the risk-reward trade-off in an incomplete market in which there is no arbitrage. As shown, perfect replication is not possible. Here, the risk-adjusted profit MR(K) for a zero profit (K=0) is greater than zero. Thus, there is some risk involved with obtaining even a zero profit (that is, there is always a risk of loss).

By contrast, the graph of FIG. 5 represents a case where the maximum risk-adjusted profit MR(K) is greater than the expected profit K. Accordingly, the deal would always yield a risk-adjusted loss. A rational portfolio manager would never undertake such a deal.

The graph of FIG. 6 represents a market where arbitrage is possible. The market may be complete or incomplete. At point K.sub.a, the risk is zero but the profit is greater than zero; thus, riskless arbitrage is possible. Nonetheless, even in this case a portfolio manager may wish to maximize the risk-adjusted profit by seeking a profit K* which bears some degree of risk (i.e., MR(K*)>0), rather than take the riskless profit of K.sub.a.

From the foregoing discussion, it will be apparent that a method and apparatus according to the present invention provides a portfolio manager with a powerful decision-making tool. Using the present invention, it is now possible for a portfolio manager to analyze the cost of a hedge in relation to a range of expected profits and risks.

FIG. 7 summarizes the relationship between risk and reward, and how this relationship relates to different types of investors. In the graph of FIG. 7, the vertical axis represents minimum regret (i.e., the cost of risk) and the horizontal axis represents the expected profit. Thus, the parabolic curve represents the cost of risk associated with a given profit that can be taken from the transaction. The point on the curve where a tangent line has a slope equal to one is the risk-neutral point, or the point where the marginal cost of risk equals marginal reward. In other words, a single unit of risk yields a single unit of reward. Given this relationship, investors who are risk-averse will opt for a degree of risk somewhere to the left of a line drawn perpendicular to the tangent, where the marginal cost of risk is less than the marginal gain in profit. That is, a single unit of risk yields more than a single unit of reward. Conversely, investors who are risk takers will opt for a degree of risk to the right of the perpendicular, where a single unit of risk yields less than a single unit of reward. An important feature of the present invention is the ability to map a relationship of the type shown in FIG. 7, thereby enabling a portfolio manager to make reasoned decisions on the desirability of undertaking a certain degree of risk.

FIG. 8 illustrates in flow chart form an exemplary method by which a portfolio manager may use the present invention to create a replicating portfolio using a computer-based system. In step 50, the portfolio manager would first supply the system with certain required input information. In this embodiment, the input information includes the composition and attributes of a target portfolio, market parameters, the instruments available for use in constructing a replicating portfolio, and any current holdings. In step 52, the system uses the input information to determine the maximum possible expected profit under the given market conditions according to equation (8) above. This step includes determining the largest expected profit K for which equation (8) is feasible. Risk is not yet taken into account. Next, in step 54, the system analyzes the risk-adjusted profit using equation (9) for decreasing levels of profit. This process stops at step 56 when a maximum risk-adjusted profit is determined; that is, the value of K that maximizes the difference between expected profit and the cost of risk (K-MR(K)). In this manner, the entire risk/reward curve may be generated.

Finally, in step 58, the system uses the previously-supplied information about available replicating instruments to generate a set of suggested trades that will produce a replicating portfolio designed to ensure the maximized risk-adjusted profit is achieved. Mathematically, the column vector x used to define the cost of risk MR(K) in equation (7) represents the trades that should be made to convert the current portfolio to a new suggested portfolio associated with the maximum risk-adjusted profit.

In yet another practical application, a system configured according to the present invention can be used to determine a risk-reward profile for a portfolio structure according to an investment strategy based on a certain stock index, such as the Standard & Poor (S & P) 500. Using the principles outlined above, the system can determine whether the investor should increase or decrease exposure to this index, or whether the investor is already at an optimal risk-adjusted position. In such an application, details of the portfolio are input to the system and used to calculate an associated measure of the regret function. A potential deal (or series of deals) is then formulated, after which an expected profit from the deal is determined. The system then determines if the risk can be decreased without decreasing the expected profit. Alternatively, the system can determine if there can be an increase in the expected profit without substantially increasing the risk.

In a risk neutral world, every security would have the same rate of return: the riskless rate of interest. Such is clearly not the case in the real world. It is therefore desirable to have some method for computing prices. A state price vector provides this benefit.

A given security can have one of a number of values at some specific future date. Those values can be represented by a matrix d, consisting of elements d.sub.ij, where d.sub.ij represents the value of the security i at the future date upon occurrence of a specific scenario j. The current value of the security can be represented by q, a known quantity. Assume further that one has perfect information, but not perfect foresight; that is, one knows the range of scenarios that could possibly occur, but not the particular scenario which will actually occur. With these assumptions, a state price vector is a set of numbers .pi..sub.1 . . . n that discount the future price of the security in a manner consistent with current prices. Stated mathematically:

q=.SIGMA..pi..sub.1 d.sub.1 +.pi..sub.2 d.sub.2 +.pi..sub.3 d.sub.3 +. . . +.pi..sub.n d.sub.n (EQ 11).

The state price vector has significant utility in the context of the present invention, with reference to the portfolio replication embodiment. Suppose a new derivative instrument is created. Although its price is not known, similar derivatives may exist in the market. For example, the new derivative (i.e., the target) may be a three-year option on the S & P, where only two-year options exist in the marketplace. In such a situation, the state price vector may be used to compute a fair price for the new instrument.

Using the above example, the present invention can produce a state price vector that will correctly price the two-year options needed to replicate a three-year option over some period less than 2 years. If the state price vector is applied to the three-year option, the present invention will obtain a price that is reasonable, without arbitrage (that is, one could not buy the three-year option and immediately sell its components at a profit). Assume for purposes of this example that one can obtain a zero

regret; that is, the three-year option can be perfectly replicated using the two-year options. Since perfect replication is assumed within the two-year period, there will be no risk. Furthermore, for all instruments in the replicating portfolio, one knows the set of numbers P that will give today's price when multiplied by the future possible values. Accordingly, for example, if one sells short the three-year option and buys a portfolio of the two-year options, at the end of the period, regardless of what occurs, the portfolio will have the same total value. Thus, one can buy back the three-year option at the end of the period and sell the replicating portfolio of two-year options and owe nothing. In short, the state price vector is the set of numbers that transform uncertain future prices in a manner that is consistent with today's price. Unfortunately, real-world markets do not contain a large enough number of instruments to permit a perfect replication.

The theory underlying the state price vector feature of the present invention can be explained as follows. A market price vector is a single vector that transforms uncertain prices at the end of a period, in a consistent manner, into prices known with certainty today.

The market price vector is represented by a non-negative vector .PSI. in R.sup.S that satisfies the following conditions:

D.PSI.=q (EQ 12)

t.sup.T .PSI.=c (EQ 13),

for an arbitrary vector t in R.sup.S. Where t is a given, non-arbitrary target vector, a vector satisfying EQ 12 and EQ 13 is referred to as a target price vector.

As before, D represents the future value of the replicating portfolio under all predetermined future scenarios and q represents the price of the replicating portfolio in today's dollars. .PSI., then, is the collection of numbers required to collapse the future, uncertain values in D into a price that is consistent with prices in today's market. .PSI. may thus be viewed as a weighted average or discounting number.

If the vector .PSI. is independent of investor preferences, the market price vector may be used to develop a set of risk-neutral probabilities. Let .rho. represent the sum of these vectors .PSI..sub.1 +.PSI..sub.2 +. . . +.PSI..sub.S. Then .omega.=.PSI..rho. may be viewed as a vector of probabilities (where .omega. has positive components that sum to 1). These probabilities are considered risk-neutral because they are independent of investor preferences.

The risk-neutral discounted present value of future payoffs must be equal to today's prices. Accordingly, equations (12) and (13) may be written as:

(.rho..sup.-1)D.omega.=q (EQ 14)

(.rho..sup.-1)t.sup.T .omega.=c (EQ 15)

Here .rho..sup.-1 represents a risk-free discount factor for the period and .omega. represents the risk-neutral probabilities.

From a practical standpoint, a portfolio manager will wish to determine whether a market or target price vector exists, whether it is unique, and the relationship between the existence of such a vector and arbitrage. The present invention employs duality theory to provide this information.

The one-norm minimum regret model used above to explain the principles of the present invention may be transformed into the following linear programming primal/dual pair.

PRIMAL:

MR(K)=Minimize p.sup.T (y.sup.+, y.sup.-) (EQ 16)

Subject to:

-y.sup.+ +y.sup.- +.rho..sup.-1 D.sup.T x=.rho..sup.-1 t; (.pi.) (EQ 17)

p.sup.T (y.sup.T -y.sup.-)-q.sup.T x.gtoreq.K-c: (.lambda.) (EQ 18)

y.sup.+ -y.sup.- .gtoreq.0 (EQ 19),

where p is some vector of scenario probabilities.

DUAL:

Maximize .rho..sup.-1 t.sup.T .pi.+(K-c).lambda. (EQ 20)

Subject to:

.rho..sup.-1 D.pi.-.lambda.q=0; (x) (EQ 21)

-p.ltoreq..pi.-.lambda.p.ltoreq.p: (y.sup.+, y.sup.-) (EQ 22)

.lambda..gtoreq.0 (EQ 23).

It can be shown that for any finite x (replicating portfolio) and arbitrary, finite t (price of target portfolio), the primal is feasible and bounded for some sufficiently small K (expected profit). Therefore, by duality theory, the dual must also be feasible and bounded.

Under constraint (17) above, the vector .pi./.lambda. represents a market or target price vector where .lambda..gtoreq.1, since .pi./.lambda..gtoreq.0. Since A may be made arbitrarily large by adjusting the expected profit K, there always exists a market/target price vector for an appropriate choice of K.

The dependence of .pi. on p (that is, the subjective preferences of market participants) arises in EQ 22. If these constraints are redundant, as they would be if y.sup.+, y.sup.- =0 (the zero regret case), then .pi. is independent of these preferences. Thus, for .lambda..gtoreq.1, .pi./.lambda. may be used to derive risk-neutral probabilities for the market, and .rho..sup.-1 multiplied by the sum of the elements of .pi./.lambda. will be the risk-free rate for the market.

Since the constraints of the dual are independent of the target t, the dual may be solved for a number of different targets with only marginally greater computational effort than is required to solve for one target alone. This observation, together with the above discussion, is the basis for using these models for pricing securities.

In most hedging situations, minimizing downside regret is often more useful than minimizing regret itself. A portfolio manager, for example, often only cares about eliminating errors that could hurt a position, not those that could enhance it.

Where only downside errors are to be minimized, the existence of a market or target price vector can be guaranteed by applying a slightly weaker requirement on .lambda.. This situation may be represented in terms of the model as follows.

PRIMAL:

MDR(K)=Minimize p.sup.T y.sup.- (EQ 24)

Subject to:

-y.sup.+ +y.sup.- +.rho..sup.-1 D.sup.T x=.rho..sup.-1 t; (.pi.) (EQ 25)

p.sup.T (y.sup.+ -y.sup.-)-q.sup.T x.gtoreq.K-c; (.lambda.) (EQ 26)

y.sup.+, y.sup.- .gtoreq.0 (EQ 27);

DUAL:

Maximize .rho..sup.-1 t.sup.T .pi.+(K-c).lambda. (EQ 28)

Subject to:

r.sup.-1 D.pi.-.lambda.q=0; (x) (EQ 29)

.lambda.p.ltoreq..pi.; (y.sup.+) (EQ 30)

.pi.-.lambda.p.ltoreq.p; (y.sup.-) (EQ 31)

.lambda..gtoreq.0 (EQ 32).

As shown, .pi..gtoreq.0 for all dual feasible solutions (from EQ 30). Since .pi.>0 if .lambda.>0, .pi./.lambda. represents a market price vector for all dual feasible solutions with .lambda.>0 (assume that p>0).

Minimizing only downside regret may be particularly advantageous for hedging purposes. According to the above constraints, EQ 30 is active if y.sup.+ >0. Thus, in such a case, .pi. is dependent on the subjective preferences of investors (i.e., .pi.=.lambda.p). The influence of investor preferences is acceptable when hedging, since hedging is the purchase of insurance based on a subjective assessment of the future. However, the ability to generate risk-neutral probabilities that are independent of investor preferences is most useful for pricing securities, and that can only be guaranteed when a minimum regret formulation yields a perfect replication with zero regret. By the above formulation, as soon as there is any regret, either upside or downside (i.e., y.sup.+ >0 or y.sup.- >0), .pi. will depend on investor preferences (p). In such cases, risk-neutral valuation is not possible. However, an analogous extension is provided by "benchmark-neutral pricing." That is, the original state price vector .pi. may be used to price any new instrument relative to a particular target or benchmark chosen.

Benchmark-neutral pricing enables one to determine a fair price for a new instrument under market conditions where perfect replication is impossible. The primal/dual equations above (i.e., EQ 16-23) still hold, since the dual constraints remain feasible and the dual solution remains optimal if the new instrument is priced "correctly." That is,

.lambda.q.sub.new =.rho..sup.-1 (D.sub.new).sup.T .pi.; (x.sub.new) (EQ 46).

The primal also remains optimal with x.sub.new =0. In light of the above, it can be seen that benchmark-neutral pricing is equivalent to risk-neutral pricing in complete markets.

The ability to detect mispricing using a market or target price vector represents a powerful feature of the present invention. Consider a situation in which one wishes to examine whether or not a security or portfolio is mispriced in a given market over some given period of time. Assume a set of future scenarios has been determined, and a set of replicating instruments with known, correct prices is available. In terms of the mathematical model, the target is the security or portfolio to be analyzed, with c representing its market price. Thus, the value of the target at the end of the period under each of the scenarios S can be represented by the components of a vector t. Using this information, a system according to the present invention can ascertain whether or not the target security or portfolio is overpriced, underpriced or fairly priced relative to the market.

Pricing analysis using the present invention is based on the observation that a rational way to view whether or not an instrument is mispriced, given the uncertainty of future events, is to examine the maximum expected risk-adjusted profit obtained when attempting to replicate it. Thus, according to the primal/dual relationship at optimality discussed above, we have:

MR(K)=.rho..sup.-1 t.sup.T .pi.+(K-c).lambda. (EQ 33).

Solving this equation for the price of the target c gives:

c=.rho..sup.-1 t.sup.T (.pi./.lambda.)+K-MR(K)/.lambda. (EQ 34).

If the target is fairly priced and there is no arbitrage, then both the expected profit and the maximum risk-adjusted profit will equal zero (that is, K=MR(K)=0) and .pi./.lambda. represents a state price vector.

As noted previously, minimum regret MR(K) and minimum downside regret MDR(K) are interchangeable in the primal dual equations depending on the purpose of the analysis. Accordingly, we choose .lambda.*=1 so that the expected risk-adjusted profit (i.e., K*-MDR(K*)) is maximized, and the price of the target may be represented as:

c=.rho..sup.-1 t.sup.T .pi.+{K*-MDR(K*)} (EQ 35).

Here, the difference K*-MDR(K*) indicates the accuracy of the price of the target. A difference that is positive, zero or negative respectively indicates the target is overpriced, fairly priced or underpriced. Thus, by straight-forward application of the principles of the present invention, a portfolio manager can make decisions concerning the price of market instruments on a more-informed basis than was previously possible.

Mispricing is possible even in a complete market (although such mispricing could not persist for long periods without the market arbitraging it away). The present invention enables determination of a true equilibrium price for an instrument even in an incomplete market.

In a complete market, there exists a zero regret primal optimal solution which satisfies the condition:

y.sup.+, y.sup.- =0; D.sup.T x=t (EQ 36).

With such a constraint, the primal reduces to:

MR(0)=Minimize 0 (EQ 37)

Subject to:

.rho..sup.-1 D.sup.T x=.rho..sup.-1 t; (.pi.) (EQ 38)

-q.sup.T x.gtoreq.K-c; (.lambda.) (EQ 39).

The dual therefore reduces to:

Maximize .rho..sup.-1 t.sup.T .pi.+(K-c).lambda. (EQ 40)

Subject to:

.rho..sup.-1 D.pi.-.lambda.q=0; (x) (EQ 41)

.lambda..gtoreq.0 (EQ 42).

A no-arbitrage situation is represented by the special case in which K=0; .lambda.>0. Note

that .lambda. is strictly positive since the no-arbitrage condition implies that for K>0 regret can no longer be zero. Since the reduced primal and dual must have equal values at the optimum, it follows that:

.rho..sup.-1 t.sup.T .pi.+(K-c).lambda.=0 (EQ 43).

Since .lambda. is positive when EQ 41 is active at the optimum, EQ 43 becomes:

.rho..sup.-1 t.sup.T (.pi./.lambda.)=c-K (EQ 44).

Here, if .pi.>0 (which is guaranteed when minimum downside regret MDR is used), .pi./.lambda. is a state price vector provided that the price of the target was adjusted to c-K. Thus, for zero regret portfolios, the expected profit K may be interpreted as the degree of mispricing in the market. As before, where K>0, the target is overpriced; where K=0, the target is fairly priced; and where K<0, the target is underpriced.

Price analysis according to the present invention conforms with logic and market observation. By definition, mispricing in a complete market can only occur in the presence of arbitrage. When there is no arbitrage, K=0 for .lambda.>0, thus:

.rho..sup.-1 t.sup.T (.pi./.lambda.)=c (EQ 45).

which indicates that the target's price must be a fair one and that .pi./.lambda. is a target price vector. Accordingly, if investors are assumed to function according to a minimum downside regret criterion, a market or target price vector will exist if and only if there is no arbitrage.

The state price vector represents an advantageous extension of the practical utility of the present invention. For example, a portfolio manager may perform a replication to identify a set of transactions which will result in an optimal risk-adjusted portfolio. Applying the duality principles discussed above, the portfolio manager may then determine a state price vector which can be used to indicate whether the instruments comprising this optimal portfolio are fairly priced. It will be readily apparent to persons skilled in the art that the present invention thus enables a portfolio manager to guard against losses with a degree of accuracy and predictability not previously possible.

In summary, given a set of financial instruments, their prices today, and their prices under all scenarios at some horizon date, a system configured according to the present invention enables a user such as a portfolio manager to compute a minimum regret portfolio (that is, one in which the cost of risk is minimized) and a corresponding state price vector. If minimum regret is zero, the state price vector produces a set of risk-neutral probabilities which may be used to compute a risk-neutral price for an arbitrary new security. If minimum regret is not zero, there is no risk-neutral price; however, a benchmark-neutral price may be computed. The approach of the present invention is constructive in that it not only produces a price, but it also produces a replicating portfolio that has that price. Such a replicating portfolio may advantageously be used as a hedge.

While the present invention has been described with reference to specific embodiments, persons skilled in the art will recognize that many modifications and variations are possible. Accordingly, the present invention is intended to cover all such modifications and variations that fall within the spirit and scope of the appended claims.

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United States Patent **6,078,904**
Rebane **June 20, 2000**

Risk direct asset allocation and risk resolved CAPM for optimally allocating investment assets in an investment portfolio

Abstract

A computer system and method for optimally allocating investment funds of an investor in a portfolio having a plurality of investments, comprising: determining a risk tolerance function for the investor specifying the investor's probability preference at each of a plurality of monetary amounts relative to a monetary range relevant to the investor, and allocating the investment funds among the investments to create an investment allocation by maximizing an expected value of a first probability density function of the investor's probability preferences determined as a function of a second probability density function of the portfolio's predicted market performance with respect to the investment funds and the investor's risk tolerance function.

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References Cited [Referenced By]

U.S. Patent Documents

5774881	Jun., 1998	Friend et al.	705/36.
5784696	Jul., 1998	Melnikoff	705/36.
5799287	Aug., 1998	Dembo	705/36.
5806049	Sep., 1998	Petruzzi	705/36.
5812987	Sep., 1998	Luskin et al.	705/36.
5812988	Sep., 1998	Sandretto	705/36.
5819237	Oct., 1998	Garman	705/36.
5852811	Dec., 1998	Atkins	705/36.
5884287	Mar., 1999	Edesess	705/36.

5911135	Jun., 1999	Atkins	705/36.

Other References

Behn, R. D., and Vaupel, J. W., "Quick Analysis for Busy Decision Makers," Basic Books, Inc., Publishers, Range of Risk Dilemmas, pp. 200-214, 1982.
Meyer, S. L., "Data Analysis for Scientists and Engineers," John Wiley & Sons, Inc., Chapter 10, Propagation of Error and Least Squares, pp. 39-40, Sep., 1992.
Papoulis, A., "Probability, Random Variables, and Stochastic Processes," McGraw Hill Book Company, Chapter 5, Sec. 5-4, pp. 151-152, Dec., 1991.
Ross, S. A., Westerfield, R. W., Jaffe, J. F., "Third Edition Corporate Finance," Irwin Series in Finance, Chapter 10, Return and Risk: The Capital-Asset-Pricing Model (CAPM), pp. 271-309, 1996.
Funds Allocation System, Product Brochure, International Business Machines, Dec. 1985.

FPS/Optimum, Product Brochure, Execution Systems Corp. 1986.
Behn, R. D., and Vaupel, J.W., "Quick Analysis for Busy Decision Makers", Basic Books, Inc., Publishers, Range of Risk Dilemmas, pp. 200-214, 1982.
Funds Allocation System, Product Brochure, International Business Machines, Dec. 1985.

FPS/Optimum, Product Brochure, Execution Systems Corp, 1986.
Schmerken, Ivy, "Making Risk Analysis Easy as Alpha, beta (BetaCalc, a Computer Program by M.D. Sass Investor Services", Wall Street Computer Review, v5, n4, p. 8(3), Jan. 1988.
Papoulis, A., "Probability, Random Variables, and Stochastic Processes", McGraw Hill Book Company, Chapter 5, Sec. 5-4, pp. 151-152, Dec. 1991.
Meyer, S. L., "Data Analysis for Scientists and Engineers", John Wiley & Sons, Inc., Chapter 10, Propogation of Error and Least Squares, pp. 39-40, Sep. 1992.
Kevles, Barbara, "Using Asset Allocation Software", Journal of Accountancy, New York, Vol. 176, Issue 5, pp. 83-87, Nov. 1993.
Reagor, Catherine, "Education of Employees on Pension Plans Urged", Business Journal --Serving Phoenix & the Valley of the Sun, v 14, n39, p. 26(1), Jul. 1994.
Glaser, Jeffrey S., "The Capital Asset Pricing Model: Risk Valuation, Judicial Interpretation, and Market Bias", Business Lawyer, vol. 50, n2, Feb. 1995.
Ross, S. A., Westerfield, R. W., Jaffe, J. F., "Third Edition Corporate Finance", Irwin Series in Finance, Chapter 10, Return and Risk: The Capital-Asset-Pricing Model (CAPM), pp. 271-309, 1996.
Mellon Bank Corporation, "Mellon Trust Announces New Products and Enhancements; New Investment Monitor Product Will Be Available Jan. 1", Dialog File 621:New Product Announcement, Oct. 1996.
Lavine, Alan, "Technology Makes it Easier to do Financial Planning (Financial Planning Software)", Accounting Today, v11, n21, p39(2), Nov. 1997.
VALIC, "VALIC Announces Customer Account Access on Web Site", Dialog File 621:New Product Announcement, Jan. 1998.

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Claims

1. A computer implemented method of allocating investment funds of an investor in a portfolio comprising a plurality of investments, comprising: determining a risk tolerance function for the investor specifying the investor's probability preference at each of the plurality of monetary amounts

relative to a monetary range relevant to the investor;

allocating the investment funds among the investments to create an investment allocation by maximizing an expected value of a first probability density function of the investor's probability preferences determined as a function of a second probability density function of the portfolio's predicted market performance with respect to the investment funds and the investor's risk tolerance function; and

allocating the investment funds comprises executing the equation: ##EQU44## where f is an allocation policy vector of N risky investments and one risk free investment;

A is a monetary amount expressed over the investor's range of potential net assets;

PP is a monetary preference probability value that quantitatively defines the investor's monetary utility for a monetary amount A;

g() is the investor's monetary risk tolerance function that relates monetary preference probability (PP) to monetary amounts;

.mu..sub.A (f) is the expected value of the investor's net assets amount as a result of implementing allocation policy f, computed from the second probability density function of the portfolio's predicted market performance;

.sigma..sub.A (f) is a standard deviation of the investor's net assets amount as a result of implementing allocation policy f, computed from the second probability density function; and

E() is an approximated expectation of PP from the first probability density function of preference probabilities obtained by mapping the second probability density function of the investor's net assets through the risk tolerance function.

2. A computer implemented method of allocating investment funds of an investor in a portfolio comprising a plurality of investments, comprising:

determining a risk tolerance function for the investor specifying the investor's probability preference at each of the plurality of monetary amounts relative to a monetary range relevant to the investor;

allocating the investment funds among the investments to create an investment allocation by maximizing an expected value of a first probability density function of the investor's probability preferences determined as a function of a second probability density function of the portfolio's predicted market performance with respect to the investment funds and the investor's risk tolerance function; and

allocating the investment funds comprises evaluating the equation: ##EQU45## where: g(A) is the investor's risk tolerance function, g(A) .epsilon. [0, 1] for A.sub.D =A<A.sub.H, and g(A.sub.D)=0, and g(A.sub.H)=1;

A.sub.D is an investor defined putative worst monetary amount;

A.sub.H is an investor defined putative best monetary amount; and

h(A.vertline..function.) is the second probability density function with respect to the investor's net asset amount given allocation policy f.

3. The computer implemented method of claim 2, wherein the investor's risk tolerance function g(A) is approximated by a truncated Taylor series about an expected value of the mapped preference probability density function.

4. The computer implemented method of claim 2, further comprising:

allocating the investment finds among the investments by selecting an investment allocation on a capital market line that maximizes E(PP) of the portfolio given the investor's risk tolerance function.

5. The computer implemented method of claim 2, wherein the investment allocation .function.* defines optimal fractional allocations of the investment funds in the investments of the portfolio, where:

.function.*=arg max E(PP.vertline..function.), .function..epsilon.CML

where:

CML is the infinite set of possible portfolios defined by the capital market line;

.function.=[.function..sub.R*,.function..sub.RF .vertline..function..sub.B] where:

.function..sub.R * is the total fractional allocation of the investment fimds to risky investments allocated optimally on the efficient frontier at the tangent point of the CML;

.function..sub.B is the total fraction of invested funds to borrow additional funds, or allocate .function..sub.RF as the total fractional allocation to risk free investments, .function..sub.RF .epsilon.[0,1] and .function..sub.RF =0 if .function..sub.B >0.

6. The computer implemented method of claim 2, wherein the investment allocation .function.* defines optimal fractional allocations of the investment funds in the investments of the portfolio, where:

.function.*=arg max E(PP.vertline..function.)

where:

.function.=[.function..sub.R.sup.T,.function..sub.RF].sup.T where:

.function..sub.R.sup.T is the fractional allocation of the investment funds to risky investments over time period T,

.function..sub.R.sup.T =[.function..sub.R,1,.function..sub.R,2, ...function..sub.R,N].sup.T .epsilon. [0,1].A-inverted.i;

.function..sub.RF is the total fractional allocation to risk free investments; and, ##EQU46##

7. A computer implemented method of allocating investment funds of an investor in a portfolio comprising a plurality of investments, comprising: determining a risk tolerance function for the investor specifying the investor's probability preference at each of the plurality of monetary amounts relative to a monetary range relevant to the investor;

allocating the investment funds among the investments to create an investment allocation by maximizing an expected value of a first probability density function of the investor's probability preferences determined as a function of a second probability density function of the portfolio's predicted market performance with respect to the investment funds and the investor's risk tolerance function;

allocating the investment funds among the investments in the portfolio by mapping the second probability density function for the portfolio through the risk tolerance function to create the first probability density function of the investor's probability preferences;

determining the expected value of the first probability density function; and

repeating the allocating step with a different allocation of the investment funds among the investments in the portfolio until the expected value of the first probability density function is maximized.

8. A computer implemented method of allocating investment funds of an investor in a portfolio comprising a plurality of investments, comprising:

determining a risk tolerance function for the investor specifying the investor's probability preference at each of the plurality of monetary amounts relative to a monetary range relevant to the investor;

allocating the investment funds among the investments to create an investment allocation by maximizing an exDected value of a first probability density function of the investor's probability preferences determined as a function of a second probability density function of the portfolio's predicted market performance with respect to the investment funds and the investor's risk tolerance function; and

wherein the investment funds include borrowed funds at a rate not necessarily equal to the risk free rate.

9. A computer implemented method of allocating investment funds of an investor in a portfolio comprising a plurality of investments, comprising:

determining a risk tolerance function for the investor specifying the investor's probability preference at each of the plurality of monetary amounts relative to a monetary range relevant to the investor;

allocating the investment funds among the investments to create an investment allocation by maximizing an expected value of a first probability density function of the investor's probability preferences determined as a function of a second probability density function of the portfolio's predicted market performance with respect to the investment funds and the investor's risk tolerance function; and

wherein the investment funds is further constrained by:

the investor's input of Value at Risk parameters A.sub.VAR and P.sub.VAR that define the maximum value A.sub.VAR the investor is willing to lose with probability P.sub.VAR ; and,

the application of these parameters to guide the solution for the overall portfolio decision vector f.

10. A computer implemented method of allocating investment funds of an investor in a portfolio comprising a plurality of investments, comprising:

determining a risk tolerance function for the investor specifying the investor's probability preference at each of the plurality of monetary amounts relative to a monetary range relevant to the investor;

allocating the investment funds among the investments to create an investment allocation by maximizing an expected value of a first probability density function of the investor's probability preferences determined as a function of a second probability density function of the portfolio's predicted market performance with respect to the investment funds and the investor's risk tolerance function; and

allocation of the investment funds to any subset of investments in the portfolio is constrained by:

an upper constraint vector .function..sub.u defining a maximum portion of the investment funds that may be allocated to the subset of investments; and,

a lower constraint vector .function..sub.l defining a minimum portion of the investment funds that may be allocated to the subset of investments.

11. A computer implemented method of allocating investment funds of an investor in a portfolio comprising a plurality of investments, comprising:

determining a risk tolerance function for the investor specifying the investor's probability preference at each of the plurality of monetary amounts relative to a monetary range relevant to the investor;

allocating the investment funds among the investments to create an investment allocation by maximizing an expected value of a first probability density function of the investor's probability preferences determined as a function of a second probability density function of the portfolio's predicted market performance with respect to the investment funds and the investor's risk tolerance function; and

allocation of the investment funds to any subset of investments in the portfolio is constrained by:

an upper constraint vector .function..sub.u defining a maximum monetary amount of the investment funds that may be allocated to the subset of investments; and,

a lower constraint vector .function..sub.l defining a minimum monetary amount of the investment funds that may be allocated to the subset of investments.

12. A computer implemented method of allocating investment funds of an investor in a portfolio comprising a plurality of investments, comprising:

determining a risk tolerance function for the investor specifving the investor's probability preference at each of the plurality of monetary amounts relative to a monetary range relevant to the investor;

allocating the investment funds among the investments to create an investment allocation by maximizing an expected value of a first probability density function of the investor's probability preferences determined as a function of a second probability densitv function of the portfolio's predicted market performance with respect to the investment funds and the investor's risk tolerance function;

receiving at least one market performance prediction for an investment period;

receiving for each market performance prediction an input weighting the market performance prediction;

combining the weighted market performance predictions to define an overall market performance prediction; and

determining the portfolio's predicted performance as a function of the overall market performance prediction.

13. The computer implemented method of claim 12, wherein:

each market performance prediction includes an interval between an upper and lower bound of expected market rates of return, and a confidence percentage defining the probability that an actual market return will be in the interval.

14. A computer implemented method of allocating investment funds of an investor in a portfolio comprising a plurality of investments, comprising:

determining a risk tolerance function for the investor specifying the investor's probability preference at each of the plurality of monetary amounts relative to a monetary range relevant to the investor;

allocating the investment funds among the investments to create an investment allocation by maximizing an expected value of a first probability density function of the investor's probability preferences determined as a function of a second probability density function of the portfolio's predicted market performance with respect to the investment funds and the investor's risk tolerance function; and

displaying for each investment:

a graphical representation of a fractional allocation of the investment funds to the investment;

a graphical representation of a user specified range for the fractional allocation of the investment funds to the investment, the range having an upper and lower bound, each individually movable to redefine the range;

displaying a graphical representation of an expected return for the portfolio given the allocation of the investment funds among the investments; and,

re-determining the expected return and the allocation of the investment funds among the investments in the portfolio in response to an input moving the upper or lower bound of the range of at least one investment so as to require a change in the allocation of the investment funds to the investment.

15. The computer implemented method of claim 14, further comprising:

displaying a graphical representation of a fraction of the investor's own funds included in the investment funds, and a user specified range for the fractional allocation of the investor's own funds to the investment funds to the investment, the range having an upper and lower bound, each individually movable to redefine the range; and,

displaying a graphical representation of a fraction of the investment funds representing investment funds that are borrowed funds as constrained by the investors upper and lower bounds.

16. The computer implemented method of claim 14, further comprising:

displaying a graphical representation of a range for the expected return, the range defined by one or more confidence intervals.

17. The computer implemented method of claim 14, further comprising:

displaying a graphical representation of a fraction of the investment funds representing investment funds invested in risk free investments.

18. A computer implemented method of allocating investment funds of an investor in a portfolio comprising a plurality of investments, comprising:

determining a risk tolerance function for the investor specifying the investor's probability preference at each of the plurality of monetary amounts relative to a monetary range relevant to the investor;

allocating the investment funds among the investments to create an investment allocation by maximizing an expected value of a first probability density function of the investor's probability preferences determined as a function of a second probability density function of the portfolio's predicted market performance with respect to the investment funds and the investor's risk tolerance function; and

determining a plurality of allocations of the investment funds in the portfolio;

determining for each allocation a predicted performance curve of the portfolio given the allocation,

the predicted performance defining for each of a plurality of predicted market performance values, a percentage of the investor's residual probability preference resulting from the allocation; and

displaying the predicted performance curves.

19. A computer implemented method of allocating investment funds of an investor in a portfolio comprising a plurality of investments, comprising:

determining a risk tolerance function for the investor specifying the investor's probability preference at each of the plurality of monetary amounts relative to a monetary range relevant to the investor;

allocating the investment funds among the investments to create an investment allocation by maximizing an expected value of a first probability density function of the investor's probability preferences determined as a function of a second probability density function of the portfolio's predicted market performance with respect to the investment funds and the investor's risk tolerance function; and

displaying on a single screen:

a first axis of probability preferences scaled between 0 and 1;

a second axis, perpendicular to the first axis, defining a range of monetary amounts including the investment funds;

the investor's risk tolerance function specifying the investor's probability preference at each of a plurality of monetary amounts;

the first probability density function for an allocation of the investment funds, the first probability density function displayed as a distribution on the first axis; and

the second probability density function for the allocation, the second probability density function displayed as a distribution on the second axis.

20. A computer implemented method of allocating investment funds of an investor in a portfolio comprising a plurality of investments, comprising:

determining a risk tolerance function for the investor specifying the investor's probability preference at each of the plurality of monetary amounts relative to a monetary range relevant to the investor;

allocating the investment funds among the investments to create an investment allocation by maximizing an expected value of a first probability density function of the investor's probability preferences determined as a function of a second probability density function of the portfolio's predicted market performance with respect to the investment funds and the investor's risk tolerance function; and

providing a plurality of reference gambles to the investor, each reference gamble defining the investor's probability preference with respect to a monetary amount.

21. The computer implemented method of claim 20, wherein providing a plurality of reference gambles comprises:

providing a question to the user, the question presenting first and second hypothetical outcomes, the outcomes equally likely, each outcome associated with a monetary amount;

receiving a user input in response to the question, the user input selected from a group including:

the first outcome;

the second outcome;

an undecided selection;

responsive to the user input being either the first outcome or second outcome:

changing the monetary amount associated with the selected outcome; and,

re-presenting the question to the user until the user input is the undecided selection.

22. The computer implemented method of claim 20, wherein each reference gamble presents first and second hypothetical outcomes, the outcomes equally likely, each outcome associated with a monetary amount, each monetary amount within a range between a first monetary amount representing a putative worst monetary amount for the investor, and second monetary amount representing a best monetary amount for the investor.

23. The computer implemented method of claim 20 wherein the plurality of reference gambles produce a plurality of paired probability preferences and monetary amounts, further comprising:

determining the investor's risk tolerance function from the plurality of reference gambles by an analytical regression over the plurality of paired probability preferences with respect to monetary amounts.

24. The computer implemented method of claim 20, wherein the plurality of reference gambles produce a plurality of paired probability preferences and monetary amounts, further comprising:

determining the investor's risk tolerance function from the plurality of reference gambles by one or more cubic splines fitting the plurality of paired probability preferences with respect to monetary amounts.

25. A computer implemented method of allocating investment funds of an investor in a portfolio comprising a plurality of investments, comprising:

determining a risk tolerance function for the investor specifying the investor's probability preference at each of the plurality of monetary amounts relative to a monetary range relevant to the investor;

allocating the investment funds among the investments to create an investment allocation by maximizing an expected value of a first probability density function of the investor's probability preferences determined as a function of a second probability density function of the portfolio's predicted market performance with respect to the investment funds and the investor's risk tolerance function; and

defining a plurality of risk tolerance functions; and

storing each risk tolerance function with data identifying the investor associated with the risk tolerance function;

displaying a plurality of risk tolerance functions;

combined selected ones of the displayed risk tolerance functions to form a mean risk tolerance function; and,

displaying the mean risk tolerance function.

26. A computer implemented method of allocating investment funds of an investor in a portfolio comprising a plurality of investments, comprising:

determining a risk tolerance function for the investor specifying the investor's probability preference at each of the plurality of monetary amounts relative to a monetary range relevant to the investor;

allocating the investment funds among the investments to create an investment allocation by maximizing an expected value of a first probability density function of the investor's probability preferences determined as a function of a second probability density function of the portfolio's predicted market performance with respect to the investment funds and the investor's risk tolerance function; and

replacing at least one of the investments from the portfolio with at least one new investment; and

re-allocating the investor's funds among the investments in the portfolio by maximizing the expected value of the first probability density function of the investor's probability preferences determined as a function of the second probability density function of the portfolio's predicted market performance with respect to the investment funds and the investor's risk tolerance function.

27. A computer implemented method of allocating investment funds of an investor in a portfolio comprising a plurality of investments, comprising:

determining a risk tolerance function for the investor specifying the investor's probability preference at each of the plurality of monetary amounts relative to a monetary range relevant to the investor;

allocating the investment funds among the investments to create an investment allocation by maximizing an expected value of a first probability density function of the investor's probability preferences determined as a function of a second probability density function of the portfolio's predicted market performance with respect to the investment funds and the investor's risk tolerance function; and

adding at least one new investment to the portfolio; and

reallocating the investor's funds among the investments in the portfolio by maximizing the expected value of the first probability density function of the investor's probability preferences determined as a function of the second probability density function of the portfolio's predicted market performance with respect to the investment funds and the investor's risk tolerance function.

28. A user interface for a computer system, for evaluating and revising an investment allocation of fractional amounts of investment funds in a portfolio including a plurality of investments, comprising:

a first axis representing a range of fractional allocation values;

a second axis including:

for each investment, a graphical representation of the fractional allocation value of the investment funds allocated to the investment;

for each investment, a graphical representation of a user specified range for the fractional allocation values of the investment funds to the investment, the range having a graphically displayed upper and lower bound, each individually movable by the user to redefine the range; and,

a graphical representation of an expected return for the portfolio given the investment allocation of the investment funds in the portfolio.

29. The user interface of claim 28, further comprising:

a graphical representation of a fraction of the investor's own funds included in the investment funds,

and a user specified range for the fractional allocation of the investor's own funds to the investment funds, the range having an upper and lower bound, each individually movable to redefine the range; and,

a graphical representation of a fraction of the investment funds that are borrowed funds (within constraints).

Description

1. MICROFICHE APPENDIX

This application includes a microfiche appendix, including 1 sheet of microfiche and a total of 36 frames.

2. BACKGROUND

2.1 Field of Invention

The invention relates generally to the field of methods and software products for financial analysis risk management, and more particularly to methods and software products for investment portfolio design and the selection, analysis of investments and the allocation of investment assets among investments.

2.2 Background of the Invention

2.2.1 CAPM & CML Background

In this section we present sufficient background of the Capital Asset Pricing Model (CAPM) and the Capital Market Line (CML) to establish the departure points for derivation of the present invention: Risk Direct Asset Allocation (RDAA) and Risk Resolved CAPM (RR/CAPM). A complete tutorial on modern asset allocation methods, particularly the CAPM and the related Arbitrage Pricing Theory, may be found in any one of a number of good texts on corporate finance [4] (A bibliography of references is found at the end of this disclosure).

The practical application of any quantitative method of portfolio design based on securities' covariance requires the selection of a `short list` of N risky stocks or other securities. Several studies have shown that the investor begins to gain "almost all the benefits of (portfolio) diversification" at N.ltoreq.8, "virtually no risk reduction" for N>15 [14], and measurable liabilities increasing beyond N=30 [15]. The nomination of the short list may be approached as a formal problem in multi-attribute utility [1]. We proceed here with a specified candidate set of N risky securities whose singular utility to the investor is their ability to contribute to a successful portfolio design.

The motivation for going beyond the CAPM, with its ever-present companion query as to "whether variance is the proper proxy for risk" [21], is in the answer that variance is only the progenitor of risk and not its final measure. Between the two there is a road, unique to each investor, to be traveled that lets us individually answer the question "how much of each of the N securities should I--not he and not she--buy and/or hold?" This question is answered by the present invention.

2.2.1.1 Risk

In the CAPM risk is measured by the rate performance dispersion of a security as expressed by its historical rate standard deviation. A primary problem with the CAPM is that once established, this `sigma` is applied uniformly to all investors independent of the amount they intend to invest or their individual aversion to the possible loss of investment assets. Thus the CAPM has a very egalitarian view of risk, and treats all investors equally, regardless of their total investment assets available for investment and net worth. The levels of risk and the concordant performance of a set of risky

securities are quantified by their covariance matrix usually computed from specified historical data.

Suppose we have a candidate portfolio of N risky securities S.sub.i, i=[1, N]. We select a past performance epoch T.sub.PE and compute the symmetrical N.times.N covariance matrix [5] for the securities as

cov S=E{(s-.mu.)(s-.mu.).sup.T } (1)

where s is the rate of return (column) N-vector of the securities and .mu. is the vector of mean or expected returns computed over a past epoch T.sub.PE where

.mu..sub.i =s.sub.i, i=1,N. (2)

We now allocate a portfolio fraction f.sub.i to each of the N securities with the elements of .function. summing to one. The total rate of return variance of such a portfolio is then given by ##EQU1## which shows the dependence of the portfolio's return variance on the allocation vector .function.. In modern portfolio theory [4] it is .sigma.(.function.) from (3) that gives the uniform measure of portfolio risk for all investors, and thus constrains CAPM to treat all investors equally.

The expected rate of return for each risky security over the investment horizon (T.sub.I) is predicted on the basis of its beta (.beta.) computed with respect to `the market` (e.g. S&P500) as follows. ##EQU2##

The familiar beta is further represented as the slope of a straight line relationship between market variation and the security in question. The frequently omitted alpha (.alpha.) parameter defines the intercept of the least squares regression line that best fits a set of security and market return rates. A method for predicting a stock's price R.sub.M from a prediction of market performance R.sub.M over T.sub.i then yields

s.sub.i =.alpha..sub.i +.beta..sub.i R.sub.M (5)

The classical CAPM formula for s.sub.i [4] generates the Security Market Line ##EQU3## where R.sub.RF is the current risk free lending rate (the historical market risk premium has been calculated at 8.5%) and R.sub.M is the expected return on the market over the investment horizon.

Keeping in mind the ability here to use other predictive security return models, in the remainder we will use the more straightforward (6) for predicting the performance of a security and understand the quoted `sigma` (standard deviation) of such a security to derive from the regression fit of K points [5] over T.sub.PE. ##EQU4##

Combining a security's expected rate of return and its standard deviation then yields the needed parameters for its assumed probability density function (p.d.f.) which fully characterizes the performance of the individual security with respect to the specified future performance of the market R.sub.M over T.sub.I.

For the RR/CAPM and RDAA developments below we additionally acknowledge an uncertain future market and express this by its variance .sigma..sub.M.sup.2 to reflect the dispersion about the predicted mean return R.sub.M. This additional uncertainty will be reflected in a given security's `sigma` to yield its total standard deviation as ##EQU5##

2.2.1.2 The Feasible and Efficient Sets

From corporate finance texts [4] we learn that a set of points termed the feasible set can be represented in 2-space where expected portfolio return R.sub.p is plotted (FIG. 1) against the standard deviation s.sub.s of the portfolio given in (3). The expected return of the `risky` portfolio allocated according to .function. is simply ##EQU6##

The efficient set is defined as the upper boundary of the feasible set drawn upward from the `minimum variance point` (MVP) since it is not reasonable to choose portfolios with the lesser expected gains for the same `risk` as measured by the portfolio's .sigma..sub.s. Therefore, according to the CAPM the optimal portfolios are all represented by the infinite set of optimal allocation vectors {.function.*} that define this upper boundary. The CAPM proceeds to resolve the problem further by introducing the risk free lending option which gives rise to the Capital Market Line.

2.2.1.3 The Capital Market Line

As shown in FIG. 1, when we introduce the risk free lending option at rate R.sub.RF, we add the (N+1)th instrument and increase the dimension of the investor's decision space to N. The CAPM argues that the optimum portfolio now lies along a line--the Capital Market Line (CML)--that originates from (0, R.sub.RF) and is tangent to the efficient set at some point E for which a unique .function.* can be discovered. Selecting a point between (0, R.sub.RF) and E defines what fraction should be invested risk free with the remainder being invested pro rata at .function.*. Points closer to E represent a larger fraction going into the risky portfolio of N stocks.

We note that the computation of the efficient set per se is not required for the solution of .function.*. As seen from FIG. 1, it is clear that if the slope of the CML is maximized within the constraints that .function. is a fraction vector whose elements sum to unity, then we would automatically obtain point E and the resulting CML. The needed slope is given by ##EQU7## where R.sub.E and .sigma..sub.E are the coordinates of E which depend on .function.. The optimal risky fraction is then obtained directly by solving the constrained non-linear optimization problem [7],[8]. ##EQU8## which yields R.sub.E *(.function.*) and .sigma..sub.E *(.function.*) from (9) and (3) respectively.

The resulting (fractional) portfolio design .function..sub.p is finally determined from ##EQU9## by appropriately selecting .function..sub.RF.

The Capital Market Line is presented as the efficient set of both risky and risk free investments and culminates the CAPM's efforts at defining a portfolio by leaving the investor with yet another infinite set of options from which to choose. At this point the CAPM simply asks the investor to apply his/her own method for picking .function..sub.RF, or as stated in [4]:

"Her position in the riskless asset, that is, the choice of where on the (CML) line she wants to be, is determined by her internal characteristics, such as her ability to tolerate risk."

The CAPM offers no guidance of any analytical method for determining each investor's allocation of investment assets on the CML.

We note that during the course of the CAPM solution there has been no discussion of actual cash amounts to be invested. The presumption being all along that, however finally obtained, the risky portfolio fractions .function..sub.R * would apply equally to billionaires and blue collar workers. This assumption thus fails to recognize that individual investors have distinct risk preferences that are intimately tied to their overall investment assets and net worth, and that as a result, would select different allocations of their investment assets.

Accordingly, it is desirable to provide a computer implemented method and software product that accounts for individual investor risk preferences as a function of the individual investor's financial profile, and thereby determines for a given portfolio of investments (i.e. short list), the optimal allocation of the investor's assets, or any portion thereof, among the investment assets.

SUMMARY OF THE INVENTION

The present invention, the Risk Direct Asset Allocation and Risk Resolved CAPM, overcomes the limitations of conventional portfolio design methods including the CAPM, and software products by determining for an individual investor that investor's risk tolerance function and selecting a monetary allocation of investment assets according to both the risk tolerance function, and quantifiable risk

dispersion characteristics of a given allocation of investment assets in the portfolio. Generally RDAA and RR/CAPM are based on integrating key elements of modem utility, securities' performance prediction, and optimization theories (see, e.g., [1,], [2], [3]) that relate to risk averse behavior in matters of monetary uncertainty.

3.1 Investor Utility and Probability Preference Curves

In accordance with one embodiment of the present invention, a risk tolerance function ("RTF") of the individual investor is determined. The risk tolerance function describes the investor's probability preferences at each of the number of monetary amounts relative to the investor's total assets. More specifically, at a given the monetary amount A, the risk tolerance function for an investor defines the probability PP(A) at which the investor is indifferent between 1) receiving the monetary amount A, or 2) accepting the risk or gamble of receiving an investor defined putative best amount A.sub.H (for `happiness` representing monetary contentment at which net worth the investor is willing to suffer essentially zero risk for further increasing his net assets) with probability PP(A) or losing his monetary assets and ending up at an investor defined putative worst amount A.sub.D (for `despair`) with probability 1-PP(A). The amounts A.sub.D and A.sub.H enclose the investor's total net current assets A.sub.T. Preferably all investment amounts and outcome calculations will be based on A.sub.T and appropriate changes to this value. Some investors may instead consider A.sub.T to be net investable assets or even their net worth. Overall then, the risk tolerance function quantitatively defines the investor's risk aversion or risk seeking behavior with respect to his unique monetary range of specified monetary amounts. Thus, the risk tolerance function is specifically scoped to the investor's actual and unique monetary range which includes his total investment assets so that it realistically quantifies the investor's preferences with respect to potential outcomes effecting the investor's assets, and hence usefully describes (i.e. quantifies probabilistically) the investor's preferences as to the market risk presented by various allocations of investment assets within a portfolio.

The investor's risk tolerance function is derived interactively in a straightforward and systematic manner through a sequence of decisions involving so-called reference gambles. Examples of several risk tolerance functions for three different investors are shown in FIG. 2. In this figure, the normalized PP value varies between 0 and 1 as the monetary outcome ranges from the investor's putative worst amount A.sub.D, to the amount of monetary contentment A.sub.H, such that PP (A.sub.D)=0 and PP(A.sub.H)=1. It is seen that the risk averse behaviors assumed here are represented by concave downward functions. The straight line joining PP(A.sub.D) and PP(A.sub.H) is the expected monetary value (EMV) line which characterizes the behavior of a risk neutral individual. Consequently the risk seeker's curve lies below the EMV line and is concave upward.

We note that the different risk tolerance functions in FIG. 2 represent different individuals as indicated. The fact that one risk tolerance function, RTF3, goes into negative territory states that this investor is willing to assume some resulting debt as the worst monetary outcome of risky investment schemes. It is reasonable, though not necessary, to assume that most mature or older investors will be risk averse with A.sub.D >0 such as in RTF1 and RTF2. All reasonable investors will exhibit A.sub.H >A.sub.T.

The monetary difference between the PP curve and EMV line at a given PP(A.sub.EMV) value is called the investor's risk premium (RP) and is seen to be the amount the investor is willing to forego or pay in order to avoid the (fair) expected value gamble at PP(A). In the figure we see that all other asset parameters given equal, Investor #1 is more risk averse than Investor #2 since RP1>RP2. Investor #3 appears to be a young person with little total assets who would be risk seeking soon after going into debt.

3.2 A General Overview of RR/CAPM and RDAA

For any given allocation of investment assets among investments in the portfolio, a probability density function can be determined which describes the rate performance dispersion of the portfolio's predicted market performance. Conventionally, this probability density function is typically

expressed with respect to a portfolio defined by fractional weightings of the investment assets, since CAPM is unable to distinguish between the risk preferences of different investors. In accordance with the present invention however, the probability density function of the portfolio's predicted market performance is expressed with respect to the investor's available investment assets, and more particularly, with respect to the investor's risk tolerance function. Thus, this probability density function describes the dispersion of potential monetary gains and losses to the investor given a specific allocation of the investor's investment assets among the portfolio. For a given probability density function, there is a mean or expected value of the probability density function. The probability density function of the portfolio, for example, describes the overall expected performance of the portfolio in monetary amounts.

In accordance with one aspect of the present invention, once the investor's monetary risk tolerance function, and the probability density function of a given investment allocation are determined, it is possible to create a probability density function of the investor's probability preferences with respect to the investor's risk tolerance function. This probability density function expresses the dispersion of risk preferences that the investor would experience as a result of the investment allocation. The expected value of this probability density function of the investor's probability preferences thus describes the overall risk preference of the investor for the specific monetary allocation of investment assets (as opposed to the conventional asset independent risk analysis).

In accordance with the present invention then, investment assets are allocated to the investments of the portfolio by maximizing the expected value of the probability density function of the investor's probability preferences. The probability density function of the investor's probability preferences is determined as a function of the probability density function of the portfolio's predicted market performance with respect to the investment assets allocation policy and the investor's risk tolerance function. The investment allocation that maximizes the expected value of the investor's probability preferences best satisfies these preferences as they are defined by the investor's risk tolerance function.

In contrast to conventional approaches, the investment allocation here describes the actual monetary amounts of the investment assets to be allocated to the investments of the portfolio. Further, because the investment allocation is determined with respect to the investor's unique risk tolerance function (s), it accounts for the investor's own particular asset base and their risk aversion or risk seeking behavior relative to such asset base. This contrasts with conventional methods that do not account for either the assets or the risk preferences of investors, and hence treat all investors as 1) having exactly the same assets; and/or 2) having exactly the same risk preferences and tolerances. For this reason, as shown above, conventional approaches based on the CAPM produce only an infinite set of potential allocations, leaving it up to the individual investor to arbitrarily allocate their actual investment assets from among the possible solutions along the CML.

The probability density function on the probability preference of the investor's risk tolerance function may be determined in a variety of manners in accordance with the present invention. In one embodiment, this probability density function is determined by numerically mapping the probability density function of the portfolio with respect to the investment assets through the investor's risk tolerance function and onto the probability preference axis. This embodiment is preferable where there is a significant probability of the investor's total assets falling below A.sub.D, the despair amount. Such an outcome is typically predicated by a large rate standard deviation for the portfolio given the investment allocation. The allocation of investment assets amongst the portfolio investments is iteratively adjusted until the expected value of the probability density function on the probability preference axis is maximized. FIG. 3 illustrates an example of the mapping of the probability density function of a given portfolio allocation through an investor's risk tolerance function onto the probability preference axis.

In an alternate embodiment, the expected value of the probability density function of the investor's probability preferences is determined by direct computation. One method of direct computation is by solution of: ##EQU10## where: g(A) is the investor's risk tolerance function, g(A).epsilon.[0, 1] for A.sub.D .ltoreq.A<A.sub.H, and g(A.sub.D)=0, and g(A.sub.H)=1;

A.sub.D is the investor defined putative worst monetary amount or `despair` amount;

A.sub.H is the investor defined putative contentment monetary amount or `happiness` amount; and,

h(A.vertline..function.) is the probability density function of the investment portfolio's predicted performance with respect to the investor's total assets given allocation policy .function..

The solution to (13) may be usefully approximated by a truncated Taylor series expansion of g(A), the investor's risk tolerance function, about the expected value of h(A.vertline..function.). One such implementation resolves (13) to: ##EQU11##

This form of the equation can be readily optimized over the selected securities for each investor to yield the actual monetary allocation over such securities to the investor's maximum expected monetary probability preference.

An examination of (14) is particularly revealing with respect to asset allocation. The first r.h.s. term is simply a mapping of .mu..sub.A onto the PP axis and is consistent with the fact that all sane RTFs are smoothly and monotonically increasing with A throughout their entire range. The second r.h.s. term is of particular interest since it adjusts the expected value of the mapped cash distribution according to two factors--the curvature of the risk tolerance function and the cash quantified standard deviation of the total portfolio both reflected in the .mu..sub.A region of investor's total assets.

We recall from FIG. 2 that risk aversion is represented by the RTF lying above the EMV line and thereby curving downward with increasing A. This translates to a negative value of the second derivative and means that a term proportional to .sigma..sub.A.sup.2 is subtracted from the direct mapping of .mu..sub.A through the RTF. We will refer to one half the RTF's second derivative evaluated at .mu..sub.A as the portfolio risk compensation coefficient (RCC). Therefore as we assume a portfolio design that increases its expected gain along the CML, we see that .sigma..sub.A also increases. Since the RTF flattens out with increasing A, the RCC becomes less negative, but the increasing .sigma..sub.A effect begins to dominate and the mapped mean, according to (14), reaches a maximum and begins decreasing at the optimal allocation point. The opposite occurs for risk seekers whose RTF falls below the EMV line in the .mu..sub.A vicinity; here the RCC is positive and the risk compensation adds to or augments the directly mapped PP value of .mu..sub.A. This rewards the investor in such a region of his anticipated total assets. Again, in the practical application of the invented algorithm and methodology to realistic short lists of stocks, the `risk seeking portfolio` at a high RCC may be characterized by high variance being traded off against a low mean because the risk seeker fully expects the high variance to work for (not against) him. We presume that current portfolio designers can take comfort from this analysis since a directly evolved form of CAPM risk as defined in Modem Portfolio Theory is very much present in the new RR/CAPM method presented here, albeit expressed in monetary (not rate) terms and mapped into the conflict resolving preference probability space.

In accordance with the present invention, the foregoing analysis and computations are embodied in a software product for controlling and configuring a computer to receive data descriptive of various investments and their risk characteristics, to interactively determine an investor's risk tolerance function, to allocate investment assets to an investment portfolio, to compute the probability density function of the portfolio's performance with respect to the investor's assets, and to compute and maximize the expected value of the probability density function of the investor's probability preferences. Additionally, the present invention may also be used in a broader context as a monetary risk management tool to determine asset allocations among sectors (e.g. large cap, bonds, growth, value, technology, metals, and the like) and also to select among candidate projects (e.g. acquire XYZ Inc., introduce product line A vs. B, buy new production facility, and the like) in a corporate planning environment.

3.3 User Interface Features

In accordance with another aspect of the present invention, there are provided various user interfaces that graphically capture and represent the investment allocation of the investment assets, along with useful information describing portfolio performance. One user interface graphically displays for each investment in the portfolio the allocation of the investment assets to the selected securities in terms of both monetary and percentage allocations, along with user definable upper and lower bounds for the allocation. There is also displayed a graphical representation of the expected return of the portfolio given the investment allocation, preferably shown with a confidence interval.

The upper and lower bounds for each investment are dynamically manipulable, and can be adjusted by the user to change the range of potential allocations to the investment. As the user moves an upper or lower bound to allow an increase or decrease in the allocation, the overall investment allocation policy among the portfolio is automatically recomputed in order to again maximize the expected value of the probability density function of the investor's probability preferences. This user interface thus allows the user to easily and dynamically manipulate the investment allocation and observe the impact of such allocations on the expected return of the portfolio.

DETAILED DESCRIPTION OF THE DRAWINGS

FIG. 1 is an illustration of the efficient frontier and Capital Market Line in the CAPM.

FIG. 2 is an illustration of example investor risk tolerance functions.

FIG. 3 is an illustration of the mapping of an investment portfolio's probability density function (translated onto the investor's total assets axis) onto the probability preference axis.

FIG. 4 is an illustration of a system in accordance with the present invention.

FIG. 5 is an illustration of the software architecture of the asset allocation program of the present invention.

FIG. 6 is an illustration of the overall data and process flow in accordance with the present invention.

FIG. 7 is an illustration of primary graphical output from the main user interface of the asset allocation program.

FIG. 8 is an illustration of a user interface for interactively editing competing scenarios of future market performance.

FIG. 9 is an illustration of a user interface for interactively generating and comparing multiple portfolio designs under uniform market conditions.

FIG. 10 is an illustration of a user interface showing a reformatted correlation matrix and related risk adjusted returns for a selected list of candidate securities.

FIG. 11 is an illustration of risk tolerance cases for creating an investor's risk tolerance function.

FIG. 12 is an illustration of a gaussian distribution of a portfolio where maximum loss is limited to the sum of risky investments.

DETAILED DESCRIPTION OF THE INVENTION

5.1 System Architecture

Referring now to FIG. 4, there is shown the configuration of a computer system in accordance with one embodiment of the present invention. The computer system environment is generally a conventional computer system that has been configured by one or more software products to operate in accordance with the methods of the present invention, to determine and provide the allocation of

investment assets to investments in a portfolio, and to output and display various user interfaces enabling the user to operate and control the system and software product.

Computer system 200 accordingly includes an addressable memory 203, and a processor 205, along with conventional input 206 (e.g. keyboard and mouse) and output (e.g. display and printers) devices 207, communication links and hard disk or other mass storage unit. The processor 205 is conventional, and executes software stored in the memory 203. The computer system 200 may be implemented with conventional hardware, such as an IBM compatible, Intel Pentium.RTM. based computer. The memory 203 stores a conventional operating system 211, such as Microsoft Corp.'s Windows 3.1 or Windows95. Also provided is a conventional network interface 213 for accessing public communications networks, such as the Internet.

Loaded into and executing from the memory 203 are components of an asset allocation program 201 in accordance with an embodiment of the present invention. Referring now to FIG. 5, the asset allocation program 201 includes an RDAA module 301, a GUI manager 305, a data handler module 307, and a communications interface gateway 308. An optional RR/CAPM module 303 may also be included. The GUI manager 305 includes a main module 309, a short list maker module 311, a portfolio editor module 313, an RTF module 315, and an account management module 310. An application executive 302 controls over all operation flow and preserves system state data.

5.1.1 Main Module

The main module 309 of the GUI manager 305 provides direct and arbitrary access to the four other user interface modules. The main module 309 provides for logon/off, and security password management to secure an investor's account data from unauthorized users. The main module 309 also allows the investor to select or nominate a current short list of investments or a portfolio for analysis by the RDAA and RR/CAPM modules 301, 303. Once the investor has logged on and selected a portfolio, the main module 309 allows arbitrary access to the other components of the system for conducting analysis of the selected portfolio.

5.1.2 RTF Module

The RTF module 315 manages an interactive dialogue with the investor to construct or edit the investor's risk tolerance function. Once the risk tolerance function is established, it is labeled, stored, and may be repeatedly accessed by the other system components. The RTF module 315 supports the following functionality:

Launch and execute interactive dialogue of reference gambles to define a new RTF.

Select and retrieve an existing RTF as working candidate.

Launch and label new working candidate RTF.

Input/edit A.sub.D `despair`, A.sub.H `contentment`, and current net asset A.sub.T amounts.

Performs both sanity and verification checks on the shape and stability of a new RTF.

Display working candidate RTF graphically.

Select multiple RTFs for comparison and display.

Request and execute RTF verification procedure by generating a series of inferred reference gamble decisions from the nominated or working candidate RTF.

Label, store/discard working selected RTF.

Nominate a stored RTF as current RTF for use by RDAA and RR/CAPM modules.

5.1.3 Short List Maker Module

The short list maker module 311 provides for selection of a number of investments for inclusion in a short list of investments to be analyzed by the RDAA or RR/CAPM modules 301, 303. In this disclosure, "investment assets" includes cash (own or borrowed) and other liquid assets that may be invested by an investor. "Investments" includes stocks, bonds, commodities, precious metals, and any other securities or financial instruments in which an investor may invest. The short list maker 311 provides the following functionality:

Establish a connection with a third party securities data service and database 319 for searching for investments.

View, nominate, and edit selection criteria for selecting an investment.

Select a current short list (SL) of investments or portfolio for analysis.

Display graphical and data tables for each candidate investment.

Access expert advice services 319 via the communications interface gateway 308.

Compute risk correlation matrix for all investments in current short list and display.

Display, edit, and/or nominate current short list of investments as new short list.

Label and store new short list and/or nominate as the current short list.

5.1.4 Portfolio Editor Module

The portfolio editor module 313 provides for selection and editing of a portfolio of investments. The portfolio editor module 313 provides the following functionality:

Select a current (default) or stored portfolio for editing.

Elect to include/exclude data for currently owned investments from an actual portfolio into the current portfolio.

Generate/edit the financial factors scenario.

Select financial factors scenario as the current scenario.

Edit limits from existing or provided default values.

Command RDAA reoptimization of current portfolio.

Edit all input amounts within specified limits.

Label/store or discard current portfolio.

Command generation of list of current buy/sell orders from the current and actual portfolios.

5.1.5 RDAA & RR/CAPM Modules

The RDAA module 301 and RR/CAPM modules 303 implement the analysis and optimization of an investor's investment allocation as set forth above with respect to (13) and (14) above, and as further explained below in .sctn.5.6. The GUI Manager 305 is responsible for presenting the outputs of the RR/CAPM and RDAA modules on the display 207 using the user interfaces of the present invention,

as variously described below with respect to FIGS. 7-10, and for obtaining user inputs for controlling the application.

5.1.6 Account Management Module

The account management module 310 provides a user interface to one or more online investment systems, such as a brokerage house to access and update an investor's account. The account management module 310 retrieves and displays securities data, corporate financials, market performance data and other research information. The account management module 310 also provides for individual trades in the investor's accounts, and transfers the list of current buy/sell order to the investor's investment account for execution.

5.1.7 Data Handler Module

The data handler module 307 includes a format and output module 317 and a file manager module 323 which communicates with a local file storage system 321. The data handler 307 manages formatting and outputting of data to the input and output devices, and retrieval and storage of data to the local file storage system 321.

5.1.8 Communications Interface Gateway

The communications interface gateway 308 provides an interface to external databases containing securities data, such as corporate financial data, industry performance, securities price and performance data, investment advisor opinions and consensus ***ratings,*** and the like, including, in some versions, more comprehensive portfolio management services without the portfolio analysis and optimization functions as provided by the present invention, as commonly available from brokerage houses, investment firms, and other sources.

5.2 The System State

The asset allocation program 201 is `stateful system` in that its internal data representation consists of a formal list of data structures and related status parameters having current values. The asset allocation program 201 performs certain functions and processes automatically and in response to user input depending on the current state of the system. The following is a list of `state variables` that are stored by the asset allocation program 201:

Current RTF: the investor's RTF that is used to calculate the optimized allocation of a given set of investment assets.

Current Short List (SL) Nominations: a set of N investments selected by the investor for optimized allocation.

Current Short List Performance: for each of the investments on the Current Short List, predicted performance parameters including a covariance matrix of market performance data.

Current Actual Portfolio: a set of investments currently held by the investor, including for each investment, a label, purchase price, purchase date, current price.

Current Allocation Constraints: investor specified limits on the percentages or dollar amounts of the investor's investment assets to be allocated to various ones or groups of investments including a Value at Risk constraint that applies to the overall portfolio.

Current Market Prediction: an estimate of market return, for example, one based on a benchmark market index, such as the S&P 500 or the Dow Jones Industrial Average.

Investment Horizon, Market Appreciation and Standard Deviation: the Investment Horizon defines the length of time for the investment; Market Appreciation is an estimate of the annualized return

during the Investment Horizon; and Standard Deviation is the standard deviation of market returns.

Current Rates: loads on individual investments, management load, tax rate, and borrowing rate.

Current Computed Portfolio: Origin--Optimized, User Input, Proportionally Recalculated. Invested own fluids amount, security labels, security amounts & portfolio fractions, borrowed amount, expected return amount and standard deviation.

5.3 System Operation: Standard User Sequence

Referring now to FIG. 6 there is shown an overall process flow of the operation of the asset allocation program 201. The process flow described herein is exemplary of a useful process flow with the system configuration outlined above. However, the system may be operated and configured in any of a number of ways to satisfy licensee requirements for their intended markets (e.g. consumer, institutional, commercial) and need not have both the risk direct and risk resolved loops.

5.3.1 Generate/Update RTF

Prior to any optimization of a portfolio, the investor creates 601 at least one RTF to define his risk preferences using the RTF module 315. Once generated the RTF is stored and accessed as needed by the RDAA and RR/CAPM modules. The investor may review and update his RTF at any time, periodically or when a financially significant event has occurred. The process of creating the investor's RTF is further described below in .sctn.5.4.

5.3.2 Generate/Retrieve Short List

Independently of establishing an RTF, the investor selects 603 a short list of candidate investments for optimization by the RDAA or RR/CAPM modules. Selection is performed via the short list maker module 311. The short list may be derived through any of a number of ways including direct input of recommended securities from experts' lists, investment advisors, or any other source. The investor may generate any number of alternate short lists, which can be individually labeled and stored for later retrieval.

For each short list, the investor specifies predicted future performance data for each investment asset. The future performance data may be the alpha, beta, sigma, R2, and cross correlations related to the Efficient Market Hypothesis approach or derived from any other predictive theory, including estimates that may by available in the securities database 319, information from investment advisors, or inputs which just reflect the investor's own assessments of the future performance of the investments. Regardless of the Market Hypothesis used, the short list maker module 311 computes and updates a covariance matrix for the short list.

5.3.3 Generate/Retrieve Financial Factors Scenario

The investor specifies 605 a scenario of financial factors that will be used to define the optimization requirements. The scenario is input and edited by the user via portfolio editor module 313. The investor inputs values for the following:

Representative epoch for securities (and market). These values define the time period over which the optimization of investment asset allocation is to be computed, and compared with market performance over the same period.

Input/edit current actual portfolio. Here the investor identifies the investments to be included in the optimization, preferably by security label, and including purchase price, purchase date, current price. The price information may be accessed and provided by the account management module 310.

Current market prediction data, including the Investment Horizon, Market Appreciation, and Standard Deviation data. Again, this data need not be manually input by the investor, but may be

extracted from existing online sources via the account management module 310.

Where individual securities are already known, the investor may input current loads on individual securities, and the investor imposed contraints on each short list candidate, owned stocks, invested, loaned (risk free), and borrowed amounts, Value at Risk. It also includes the buy/sell and portfolio management fees, tax rates, and purchase cost, etc. of owned stocks. Specifically for the stocks it includes the alfas, betas, sigmas, correlation coefficients, valid data epochs (may be different for each stock), for the stocks and the market predictions from FIG. 8.

Finally, the investor labels the Financial Factors Scenario for storage and later retrieval.

5.3.4 Generate Optimized Portfolio

The investor generates 607, 609 an optimized allocation of investment assets for the current short list, including owned securities, of investment assets, using the RDAA module 301 and for the RR/CAPM module 303. The optimized allocation specifies the dollar amount to be invested in each of the short list investments to achieve the optimized risk reduced investment return. To initiate the optimization, the investor selects a current short list and its related Financial Factors Scenario, and one of the optimization modules 301, 303. The selected module verifies that a complete dataset exists for portfolio computation. The optimization module then computes and outputs for display the newly computed investment allocation. The computed investment allocation, and accompanying short list and financial factors is labeled and stored for later retrieval and if desired, editing. If the short list or financial factors scenario changes, then the investment allocation needs to be recomputed.

5.3.5 Edit & Review Computer Portfolio

Once an investment allocation for a portfolio is computed, the investor may review and edit 611, 613 any of the variables defining the portfolio using the portfolio editor module 313. The investor selects either the current portfolio for editing or a stored portfolio for retrieval. The following variables may then be edited:

Input data on any original short list member (whether included in portfolio or not).

Edit short list securities' investment constraints and/or amounts.

Edit own investment assets constraints and/or amount.

Specify of new candidate investments for the short list, including obtaining new candidate investment performance statistics, testing a new candidate investment in the current portfolio design; launching a query into the securities database 319 to find specified investment candidates; or launching an agent into Internet to find a specified candidate.

Edit borrowed investment asset constraints and/or amount.

Interrogate predicted portfolio performance through editing portfolio return confidence intervals.

Edit securities' predicted performance parameters.

Following editing, the investor then commands recomputation or reoptimization of the portfolio to produce a revised computed portfolio which itself may be further edited or discarded or labeled and stored.

5.3.6 Access Brokerage Services

With one of the computed portfolios, the investor then accesses his online investment account via the account management module 310. The account management module 310 automatically compares the current actual portfolio with the newly generated portfolio, generates an editable buy/sell table which,

upon execution 615, changes the investor's actual portfolio to match the computed portfolio. Depending on the facility of the online brokerage service, the investor may issue a comprehensive buy/sell order or use the generated buy/sell table to place the individual orders as allowed by the service and obtain an updated actual portfolio.

5.4 User Interface Features

In the form of annotated screen designs in selected figures, the present invention provides a number of new user interfaces for editing and understanding the performance and risk characteristics of a given portfolio. The user interface displays are as follows:

5.4.1 Portfolio Design

Referring now to FIG. 7, there is shown an example of an interactive user interface which provides for the primary output of the asset allocation program 201. The portfolio design screen 701 summarizes what investment funds are invested (own 729 and new SL candidates (2-10)), under what constraints they are to be allocated (the bold brackets 707), the actual monetary amounts which the RDAA module 301 or RR/CAPM module 303 allocated to the risky and risk free vehicles (ten securities are shown with their ticker labels 713 indicated along with the risk free amounts 727), and finally the risk compensated portfolio's predicted performance 717. Additional data, such as the computed PP value of the current design and dollar amounts may also be included.

The portfolio design screen 701 is constructed as follows. Along the X-axis are listed each of the investments 711 included in the computed portfolio, each investment listed by its corresponding label 713. Also listed are columns representing risk free funds 727, the investor's total available own investment funds 729, and borrowed investment finds 731. For each of the investments 711, a monetary amount invested in the investment is shown in bar format as a bar 709 with respect to two Y-axis scales. A first Y-axis 703 is scaled as percentages of the investor's total investment funds (equal to the investor's own funds 729 and the borrowed funds 731). A second Y-axis 705 is scaled in currency amounts. Thus for each investment 711 or investment funds 727, 729, 731, the amount to be invested according to the RDAA module 301 is directly shown.

The portfolio's predicted performance is displayed by the rightmost bar 723 and bracket 735. The height of the bar 723 indicates the expected cash return during the investment period; the percent return 717 is also indicated as a percentage of the investment funds. The bracket 735 displays the symmetrical performance uncertainty within a confidence window 719 into which the portfolio's return will fall. In this example, the optimized portfolio will yield an expected return of 12% and with 90% certainty the portfolio's return will fall between a maximum of about 35% and a minimum of about 5%. The overall likelihood of having a negative return (i.e. losing money) is the indicated 9%.

Accompanying each bar 709 is a constraint bracket 707. The upper and lower handles of the constraint bracket 707 are adjustable by the investor (as shown by arrows 715) via the mouse 206 to define the maximum and minimum percentages 703 or amounts 705 to invest (left Y-axis 703) and the absolute monetary amounts and or limits (right Y-axis 705) of own and borrowed investment funds to invest. Likewise, the actual investment amount 709 is also adjustable by the investor as shown by arrows 718. When the investor adjusts such an amount directly, the algorithm takes that as an equality constraint, essentially collapsing the maximum and minimum brackets to the indicated amount, and computes the optimum within the remaining degrees of freedom on other portfolio parameters. Proceeding in this manner, the investor adjusts the constraint brackets 707 and/or the bar 709 for the individual investments 711 and funds 727, 729, 731 to define a particular financial factors scenario. When these inputs are complete, then RDAA module 301 is commanded to recompute and display the optimal portfolio.

The portfolio design screen 701 may also be assymetrically defined by moving the appropriate crossbar and the display will dynamically update the optimal portfolio solution with the new confidence probability. The lower rounded box 721 always indicates the probability (here 9%) that the portfolio will result in actual reduction of investor's current investment funds. The upper rounded

box 717 always indicates the expected portfolio return (here 12%) under the current portfolio design and input financial factors scenario.

From this display 701 the investor may now begin to examine the sensitivity of a given portfolio's risk and return by reconstituting the investment and invested amounts by appropriately changing the size of the bars 709 and constraint brackets 707 on the display 701. In response to an adjustment, the asset allocation program 201 dynamically recomputes the portfolio's performance and continuously updates the display 701. If the investor exactly specifies all invested amounts, then the use of such forced inputs directly calculates the resulting monetary utility PP and does not require numerical optimization. Such a responsive display gives the investor unparalleled insight about how the portfolio responds to different allocation policies. In the final analysis the investor may, of course, opt for a slightly non-optimal portfolio that may satisfy some other non-quantifiable subjective criteria which still produces an acceptable return and risk which he naturally intuits (and has corroborated by the updated PP value display). It should be noted that, the RDAA solution is still optimum within the investor imposed constraints. It should be clear that the less constraints the investor imposes, the higher the PP value (monetary utility) the RDAA module 301 is able to obtain from its optimization. Investor constraints simply reduce the possible maximum PP value.

5.4.2 Multiple R.sub.M integration

Referring now to FIG. 8 there is shown an example screen display 801 for interactively inputting and editing four competing predictions of future market performance given by, for example, an Investment Advisor (indicated as IA), two expert sources (E1, E2), and the investor (Self). The predictions are given in the bracket formats usually obtainable from the investment information sources, such as securities database 319. Each bracket 803 has an associated confidence percentage 805 which represents the probability that the actual market return 807 will be in the indicated interval, and may or may not have an indicated mean 811 and average confidence percentage 813. The investor is able to make the adjustments where indicated by the up/down arrows 809; he may also type in the amounts by selecting the on-screen number (e.g. percentage 805, return 807) which is then boxed as shown for the IA. Lastly, the investor can adjust a relative weight 817 for each input by moving the top of each weighting bar 817 as indicated by arrows 815.

The asset allocation program 201 responds by dynamically recomputing and continuously updating the display of the "Weighted Average" market response 811. The investor may also individually move either end bar to obtain the display of the probability that the composite market return falls in the indicated window. The RDAA and RR/CAPM modules may then be commanded to use the recomputed average and its related standard deviation to recalculate the optimal investment allocations for any selected portfolio.

The final market prediction may be deleted, and/or named and saved for further use as part of the current financial factors scenario. From this part of the application it will also be possible to recall other such screens of previously developed market performance predictions and nominate them for further use.

5.4.3 Competing Portfolios as a Function of R.sub.M and Relative Performance of Portfolios under Different Market Performance ***Futures***.

Since the asset allocation program 201 permits the investor to generate and store several portfolio designs (both optimal and sub-optimal) as described above, it is important to provide him an opportunity to compare the relative performance under uniform market conditions. FIG. 9 illustrates an interactive screen 901 for performing this task.

In this screen display 901, for each of several investor selected portfolios P1, P2, P3, and P4, the performance 903 of that portfolio is plotted against the investor's residual PP value as a function of expected market performance 917 (holding standard deviation constant) at the investment horizon. The vertical axis 903 is scaled in the percent of PP remaining (i.e. 1-PP(A.sub.T) which is a meaningful comparator to the investor since it is indexed from his current asset (A.sub.T) level. The

horizontal axis 905 is scaled to expected percent market returns.

The downward shape of the curves P1-P4 indicates that the investor is in the risk averse part of his RTF since the marginal PP gain drops for every additional increment of market return. Generally it is clear that Px is better than Py if and only if Px>=Py over the range of anticipated market performance, however FIG. 9 has no such pair of curves. Rather, FIG. 9 shows the more realistic and difficult situation where some portfolios, eg. P1 and P2, do well with high market performance values but do poorly faster when the market goes down (holding the standard deviation constant) as compared to P3 and P4. Portfolio designs P3 and P4 may be considered more `defensive` and therefore reasonable if there is a significant likelihood of poor market performance. However, the RDAA or RR/CAPM modules already account for any given market performance p.d.f., and thus FIG. 9 shows only the sensitivity to variation in the mean value.

The investor may select a prestored market scenario 907 (here MS#2) from a pull down list from the indicated label. The investor then modifies the shaded confidence boundaries 911 of the confidence interval 918 as indicated by the left/right arrows 913 to define a new confidence interval 918 for the market performance until the investment horizon. The asset allocation program 201 immediately displays the confidence level 915 (here 90%) that the current market scenario return will fall within the new shaded area.

Finally, the investor can view dynamically updated portfolio performance curves Pi by changing the mode of this display to accept as input dragging the expected market return line 917 to a new value (as shown by arrows 919) and adjusting the now symmetrical width of the confidence interval 918 to the confidence level that reflects the new uncertainty in the market's performance. This dynamic, interactive display again provides novel and valuable insight to the sensitivity of the performance comparisons for the competing portfolios.

5.4.4 Correlation Matrix

Referring now to FIG. 10 there is shown another screen display 1000 of the present invention, this one representing a modified correlation matrix of risk adjusted returns for a portfolio of candidate securities. The risk adjusted returns may be conventionally calculated; the present invention here provides a new and more insightful way of understanding the standard correlation matrix.

In FIG. 10, the risk adjusted returns--expected return divided by the standard deviation--for each investment 1007 as shown on the main diagonal 1001 as varying height rectangular boxes 1003, and the correlation coefficients for all pairs of securities are shown at the intersecting off-diagonals.

The lines 1005 above the wider rectangles of the risk adjusted returns indicate the marginal returns available if the predicted market appreciation were perfectly known--i.e. if .sigma..sub.M =0--and therefore quantify the decrements due to market uncertainty as another function of our ignorance of the future. The returns and intersecting correlation coefficients of the investments selected are displayed by the filled rectangles with the rejected investments indicated as outlined rectangles.

In the asset allocation program 201, this screen display 1000 enables the investor/analyst to quickly get the `feel` for how RDAA module 301 determines portfolio membership. This feedback is valuable to the investor as he adds new investments to a portfolio that might provide additional diversification benefits. From the figure we notice that the selected investments (Nos. 1, 3, 6 and 8) all have relatively high risk adjusted gains and low cross correlation values. The largest sum is invested in security #8 which itself is unique in the short list because it has an almost uniformly low performance correlation with all the other stocks thereby providing the most diversification value in its inclusion.

5.5 RTF Module

As described above, the RTF module 315 is responsible for providing an interactive dialogue to establish (abstract and quantify) an investor's risk tolerance function (or multiple functions). The RTF

module 315 provides the interactive dialog in terms of 50/50 reference gambles, detects and filters inflections above/below the EMV line, and creates an RTF either by appropriate candidate analytical functions and/or splines.

Table 1 presents the nominal sequence of reference gambles that the investor is asked to resolve to obtain the points in the A-PP space defined over the total assets line by the investor's A.sub.D, A.sub.T, A,.sub.H values. It is these captured points {X.sub.i, PP(X.sub.i)} which are used to fit the analytical RTF in either single function regression or in cubic spline format. Here the X refers to the cash amounts used in the reference gambles. The RTF module 315 begins by accepting investor inputs of their A.sub.D, A.sub.H, and A.sub.T amounts.

TABLE 1

Sequential Capture of Risk Tolerance Function Points

RG #	$ High	PP(High)	$ Low	PP(Low)	$X (input)	PP(X)
1	A.sub.H	1.0	A.sub.D	0	X.sub.1	0.5
2	X.sub.1	0.5	A.sub.D	0	X.sub.2	0.25
3	A.sub.H	1.0	X.sub.1	0.5	X.sub.3	0.75
4	A.sub.H	1.0	X.sub.3	0.75	X.sub.4	0.875
5	X.sub.3	0.75	X.sub.1	0.5	X.sub.5	0.625
6	X.sub.1	0.5	X.sub.2	0.25	X.sub.6	0.375
7	X.sub.2	0.25	A.sub.D	0	X.sub.7	0.125

Table 1 is explained as follows. At the start the investor is presented with a choice of 1) taking a certain, perhaps negative, $X.sub.1, thus making his total assets A.sub.T +X.sub.1, or 2) choosing a 50/50 gamble (i.e. toss of fair coin) where winning yields (A.sub.H -A.sub.T) thus bringing his total asset to A.sub.H. Losing the gamble results in a dollar loss of (A.sub.T -A.sub.D) thereby reducing the total assets to A.sub.D. Starting with an arbitrary value, X.sub.1 is increased if the gamble is chosen and decreased if the certain X.sub.1 is chosen. (The process is speeded up if the initial

X.sub.1 is such as to keep A.sub.T +X.sub.1, near A.sub.T.) In this manner the investor is quickly driven to the point of indifference or indecision, both indicating that the two alternatives are of equivalent preference. From the computation of the decision graph this becomes the (0.5, A.sub.T +X.sub.1 point on the RTF where PP(A.sub.T +X.sub.1)=0.5=[0.5.times.0+0.5.times.1.0].

The A.sub.T +X.sub.2 0.25) point is determined by asking the investor to choose between 1) taking the certain, perhaps negative, amount X.sub.2, thus making his total assets A.sub.T +X.sub.1 +X.sub.2, or 2) choosing the 50/50 gamble where winning now yields A.sub.T +X.sub.1 -X.sub.2 and losing yields a loss of A.sub.T +X.sub.1 -A.sub.D bringing his total asset to A.sub.D. Again X.sub.2 is raised if the gamble is chosen and vice versa thus bringing the investor to a point of equivalent preference between the presented alternatives. Since the best outcome now had a PP=0.5 (and the worst was, of course, PP(A.sub.D)=0) from the first reference gamble, the current preference probability PP(A.sub.T +X.sub.1 +X.sub.2) is assigned the value 0.25=[0.5.times.0+0.5.times.0.5].

We may now split the 0.5 to 1.0 PP interval to determine the (A.sub.T +X.sub.1 +X.sub.3, 0.75) point by asking the investor to choose between 1) getting the certain amount X.sub.3 added to his now total assets of A.sub.T +X.sub.1, or 2) taking a 50/50 gamble where winning will bring his total assets to A.sub.H and losing will maintain them at A.sub.T +X.sub.1. Again X.sub.3 is raised if the gamble is chosen and vice versa until equivalent preference is reached. The value of PP(A.sub.T +X.sub.1 +X.sub.3) is 0.75=[0.5.times.1.0+0.5.times.0.5].

In similar ways we may continue splitting the PP intervals until the points yield an unambiguously smooth track between (0, A.sub.D) and (1, A.sub.H). About four or five points are usually needed to accomplish this. During optimization a smooth analytical regression function is dynamically fitted to a subset of these points over the monetary `region of activity` to yield the desired RTF that is used in the portfolio design methods below.

The present invention avoids the oft-cited circular logic (preference conflicts) and monetary relevance pitfalls of utility theory in such applications by presenting the reference gambles only within the investor's relevant monetary spectrum as described above, and by detecting `insane` PP points and automatically reinterrogating that portion of the monetary axis until a `sane` RTF is obtained. In practice this requires at most two passes. Insane PP points may be detected as RTFs having more than one inflection point.

That making monetary decisions, which are based on an individual's (or corporation's) RTF, is both a rational and practical approach is summarized from Behn and Vaupel [3] as:

1. Preference probabilities have a precise definition and can therefore be explained and captured unambiguously. Once A.sub.D and A.sub.H are given, the meaning of any mediating PP is clear. If, for example, an investor's RTF goes through the point PP=0.7 and $1,000,000, this states that the investor is indifferent between receiving $1,000,000 for certain and a gamble that will yield her A.sub.H with probability 0.7 or A.sub.D with probability 0.3. Even though both contingencies are hypothetical, the investor has been able to decide a priori that the value of the certain monetary outcome and the gamble are identical in her monetary spectrum.

2. Decision dilemmas can be readily resolved by assessing the PPs for each outcome and choosing the outcome with the highest PP. This is possible because each outcome can be replaced in the decision graph with its equivalent reference gamble. Applying the probability rules for computing the graph makes it clear that the investor will then prefer the decision which approaches the best outcome A.sub.H, with the highest probability.

3. As an adjunct to the present development, the assignment of PPs to outcomes through reference gambles is applicable for all sorts of decisions "whether the consequences . . . are wealth, health, or happiness." Modern utility theory recommends that a decision maker can use PPs to assess the relative preference of each contemplated outcome.

To this list the present invention adds the important fourth reason in that such piecewise analytical

expression of an investor's RTF over the monetary domain of interest makes possible the efficient application of the RR/CAPM and RDAA methods described herein. Prior to the development of this practical approach, monetary utility has been defined over arbitrary regions, such as zero to infinite dollars, disconnected from any actual investor, and consequently has experienced a controversial relationship to quantitative portfolio design only in academic discussions.

5.5.1 Example Interactive Dialogue for Capturing Risk Tolerance Function

In one implementation, the following cases are examples used by the RTF module 315 to define an investor's RTF. Their order of occurrence is somewhat arbitrary after the first reference gamble. Each case will have an intrinsic PP associated with it. Throughout we maintain A.sub.D .ltoreq.$L<$H.ltoreq.A.sub.H and A.sub.D <A.sub.T <A.sub.H. Each case X (X=1 . . . 6) continues offering the contingencies in paragraphs X.1 and X.2 under each case X described below until the investor branches to X.3 (indifference) at which time the next case in order is introduced. This process continues until all the specified cases are completed thus yielding a complete set of investor's RTF points. If the investor delays a long time (e.g. 30 seconds) before deciding (as described in the X.4 paragraphs), the RTF module 315 automatically suggests to the investor that he may have reached his indifference point and thus may be able to chose case X.3. The X-numbered cases are illustrated in FIG. 11. When each dialog is presented to the investor, all variables are instantiated and formulas computed to present the actual dollar amounts in the dialog itself. The questions which present the contingencies are merely exemplary, and other hypotheticals may certainly be used.

The first gamble always determines the PP=0.5 indifference dollar amount that will split the A.sub.D to A.sub.H interval. The subsequent gambles will continue splitting the so generated intervals iteratively while assigning the middle values of PP to the median indifference amounts as obtained from the cases below. It is clear then that the second gamble may be chosen to determine either the 0.25 or 0.75 PP points. These PP intervals may be further split in an arbitrary order. It is almost always the case that obtaining the cash amounts for the intervening PP points 0.5, 0.25, 0.125, 0.375, 0.75, 0.625, 0.875 is sufficient to determine the most complex RTF. The cases illustrated below cover the spectrum of possible reference gambles to obtain the dollar amounts to an arbitrary resolution of the RTF.

We note that such a detailed dialogue will most likely be used only by the new investor and that it will be replaced by direct manipulation of the monetary reference gambles for the experienced investor who will no longer need the type of `introductory story line` presented below. The `direct mode` will be selectable within the RTF module.

Case 1. (AD.ltoreq.$L<$H<AT)

"You have been involved in an unfortunate lawsuit and the jury will certainly deliver one of two equally likely judgments against you if you do not agree settle the suit. The worst judgment requires you to pay the amount W=(AT-$L), and in the best outcome you must pay B=(AT-$H). The plaintiff's attorney offers to settle for the amount S=(AT-($L+$H)/2), would you pay the demand or go ahead with the jury trial?"

1.1 "PAY DEMAND"

"Before you reply the demand is peremptorily raised to S=(S+W)/2, would you pay this new amount?"

1.2 "JURY TRIAL"

"Before you reply the demand is peremptorily reduced to S=(S+B)/2, would you pay this new amount?"

1.3 "DON'T CARE or CAN'T DECIDE"

"Thank you. Now let's go on."The RTF module 315 records (AT-S) and its PP as a point on the investor's RTF.

1.4 Long Delay

"If it is difficult for you to decide, then you may have reached the point where you are indifferent to paying the demand or letting the matter go to trial. For estimating your tolerance to monetary risk this indicates that you indeed can't decide or don't care which path you take."

Case 2. (AD.ltoreq.$L<AT) & ($H=AT)

"Your hillside house will suffer mudslide damage to the tune of W=(AT-$L) if you do not build an engineered retaining wall for the bid price of S=W/2. Reliable weather data for the coming season makes it equally likely that your house would be damaged or undamaged if you don't build the wall. Would you build the retaining wall?" (here B=0)

2.1 "BUILD WALL"

"Before you can reply to the bid the contractor notifies you that the cost of the wall has increased to S=(S+W)/2, would you still build the wall?"

2.2 "DON'T BUILD WALL"

"Before you can reply to the bid the contractor notifies you that the cost of the wall has been decreased to S=(S+B)/2, would you now build the wall? "

2.3 "DON'T CARE or CAN'T DECIDE"

"Thank you. Now let's go on."

Record (AT-S) and its PP.

2.4 Long Delay

"If it is difficult for you to decide, then you may have reached the cost point where you are indifferent to protecting your house or taking a chance with the weather. For estimating your tolerance to monetary risk this indicates that you indeed can't decide or don't care which path you take."

Case 3. ((AD.ltoreq.$L<AT) & (AT<$H.ltoreq.AH)) & ((AT-$L).gtoreq.($H-AT))

"You paid W=(AT-$L) to enter into a speculative project which it now appears will reward you less than hoped for at best. In fact, it is equally likely that you may be able to sell out for a profit of B=($H-AT) or be stuck with the loss of W. A third party suddenly offers to buy your share for S=(W-X). Would you take the guaranteed amount from the third party or go ahead with the risky sale when (X=(AT-($L+$H)/2)?"

3.1 "ACCEPT BUY OUT"

"Before you can reply to the third party you are notified that the buy out price is reduced to S=S/2, would you still accept the buy out?"

3.2 "GO AHEAD WITH SALE"

"Before you can reply to the third party you are notified that the buy out price has been increased to S=(S+B)/2, would you still accept the buy out? "

3.3 "DON'T CARE or CAN'T DECIDE"

"Thank you. Now let's go on."

Record S and its PP.

3.4 Long Delay

"If it is difficult for you to decide, then you may have reached the cost point where you are indifferent to selling out your stake (or accepting a loss) for a certain amount, or taking a chance with the uncertain sale. For estimating your tolerance to monetary risk this could mean that you indeed can't decide or don't care which path you take."

Case 4. ((AD.ltoreq.$L<AT) & (AT<$H.ltoreq.AH)) & ((AT-$L).ltoreq.($H-AT))

"You paid W=(AT-$L) to enter into a speculative project which it now appears may also reward you nothing. In fact, it is equally likely that you may be able to sell out for a profit of B=($H-AT) or be stuck with the loss of W. A third party suddenly offers to buy your share for S=(X-W). Would you take the guaranteed amount from the third party or go ahead with the risky sale when X=(-AT+ ($L+$H)/2)?"

4.1 "ACCEPT BUY OUT"

"Before you can reply to the third party you are notified that the buy out price is reduced to S=S/2, would you still accept the buy out?"

4.2 "GO AHEAD WITH SALE"

"Before you can reply to the third party you are notified that the buy out price has been increased to S=(S+B)/2, would you still accept the buy out? "

4.3 "DON'T CARE or CAN'T DECIDE"

"Thank you. Now let's go on."

Record S and its PP.

4.4 Long Delay

"If it is difficult for you to decide, then you may have reached the cost point where you are indifferent to selling out your stake for a certain amount or taking a chance with the uncertain sale. For estimating your tolerance to monetary risk this could mean that you indeed can't decide or don't care which path you take."

Case 5. ($L=AT) & (AT<$H.ltoreq.AH)

"You are in a fortunate position of having the option to enter into a project that can net you either B= ($H-AT) or at worst cost you nothing (W=0). In fact reliable odds are that either case is equally likely. The inevitable third party approaches and offers to buy your option for S=($H-AT)/2. Would you still go ahead with the project or sell your option?"

5.1 "ACCEPT BUY OUT"

"Before you can reply to the third party you are notified that the buy out price is reduced to S=S/2, would you still accept the buy out?"

5.2 "GO AHEAD WITH PROJECT"

"Before you can reply to the third party you are notified that the buy out price has been increased to S (S+B)/2, would you still accept the buy out? "

5.3 "DON'T CARE or CAN'T DECIDE"

"Thank you. Now let's go on."

Record S and its PP.

5.4 Lone Delay

"If it is difficult for you to decide, then you may have reached the cost point where you are indifferent to selling your option for a certain amount or taking a chance with the uncertain project. For estimating your tolerance to monetary risk this could mean that you indeed can't decide or don't care which path you take."

Case 6. (AT.ltoreq.$L) & ($H.ltoreq.AH)

"You are in the most fortunate position of having the option to enter into a project that can net you either B=($H-AT) at best, or at worst still net you W=($L-AT). In fact reliable odds are that either case is equally likely. The inevitable third party approaches and offers to buy your option for S=(B+W)/2. Would you still go ahead with the project or sell your option?"

6.1 "ACCEPT BUY OUT"

"Before you can reply to the third party you are notified that the buy out price is reduced to S=(S+W)/2, would you still accept the buy out?"

6.2 "GO AHEAD WITH PROJECT"

"Before you can reply to the third party you are notified that the buy out price has been increased to S=(S+B)/2, would you still accept the buy out? "

6.3 "DON'T CARE or CAN'T DECIDE"

"Thank you. Now let's go on."

Record S and its PP.

6.4 Long Delay

"If it is difficult for you to decide, then you may have reached the cost point where you are indifferent to selling your option for a certain amount or taking a chance with the uncertain project. For estimating your tolerance to monetary risk this could mean that you indeed can't decide or don't care which path you take."

5.6 Implementation of Risk Resolved CAPM by the RR/CAPM Module

We now turn to a detailed discussion of the operation of the RR/CAPM module 303 and its computation of an optimized investment allocation. The operation of the RR/CAPM module 303 is to select the dollar optimal point on the CML as shown in FIG. 1 and defined by the solution to (11). CAPM yielded the fractional portfolio design .function.* given in (11) with .function..sub.RF as the yet to be determined free parameter.

We begin by noting that the CML can be quantified to define the expected monetary value of an investor's total assets as

.mu..sub.AF (.function..sub.RF)=A.sub.T +A.sub.1 [.function..sub.RF R.sub.RF +(1-.function..sub.RF)R.sub.E (.function.*)] (15)

that is the amount-mapped CML conditioned on .function..sub.RF .epsilon.[1,0] where R.sub.E is the expected rate of return from the risky (point E) portfolio now allocated according to the fixed .function..sub.R *. A.sub.1 is the amount to be invested and may be made up of funds drawn from A.sub.T and borrowed at R.sub.RF. We apply (15) in the case when A.sub.1 A.sub.1T, an amount entirely drawn from A.sub.T.

If additional funds A.sub.B =.function..sub.B A.sub.1T are borrowed, then the CAPM prescribes that .function..sub.RF =0 and the optimum portfolio gain is somewhere beyond point E wifh portfolio consisting entirely of risky investments allocated according to .function..sub.R *. Assuming that the investor uses some fuids A.sub.1T from A.sub.T, we may condition the total assets in this case on the fraction .function..sub.B >0 of A.sub.1T borrowed as

.mu..sub.AB (.function..sub.B)=A.sub.T +A.sub.1T {R.sub.E (.function.*)+.function..sub.B [R.sub.E (.function.*)-R.sub.RF]}(16)

The cash standard deviation of the `lend only` portfolio is seen to be

.sigma..sub.ARF =A.sub.1 (1-.function..sub.RF).sigma..sub.E (17)

where .sigma..sub.E =.sigma..sub.S (.function..sub.R *) from (3). It is clear how this classical expression of portfolio risk diminishes as a larger fraction is invested at R.sub.RF. The equivalent dispersion for the `borrow only` portfolio is

.sigma..sub.AB =A.sub.1T (1+.function..sub.B).sigma..sub.E (18)

which is seen to increase appropriately as more borrowed funds are invested. Since A.sub.1T =A.sub.1 at .function..sub.RF =.function..sub.B =0, the risks are equivalent at the E point as expected.

The basic premise behind RR/CAPM and RDAA is that the investor makes monetary choices in uncertain situations according to the investor's specific tolerance for risk as was discussed in .sctn.5.5. Therefore, the salient monetary decisions among alternative portfolios are to be resolved so as to maximize the expected value of PP on the investor's RTF. This is in direct opposition to the conventional view of attempting to represent the risk measure in such terms as the classical `risk adjusted return` for an investment computed as its expected rate of return divided by the standard deviation of that rate. The conventional commensurate measure for a portfolio would then be given from (4) and (10) by the ratio R.sub.p (.function.)/.sigma..sub.s (.function.). This further demonstrates how the conventional approach ignores individual attitudes toward monetary risk.

The RR/CAPM approach is based on individual risk tolerance expressed over a bounded and currently relevant monetary spectrum. It takes the probability density function (p.d.f.) of predicted total assets at the end of the investment horizon, as defined by (15) through (18), and maps this onto the individual's PP values as represented by the RTF. This mapping is shown in FIG. 3. Specifically we seek to compute the mean of the mapped distribution on the PP axis given by E (PP.vertline..function.) where .function. is now the appropriate portfolio design fraction vector in the sense discussed above. Let the RTF be represented by the analytical regression g(A) .epsilon.[0,1] for A.sub.D .ltoreq.A.ltoreq.A.sub.H with all needed derivatives and where g(A.sub.D)=0 and g (A.sub.H)=1. In practice, g(A) need only be locally analytical in the sense described above. Then ##EQU12## where h(A.vertline..function.)is the portfolio's p.d.f. on total assets. Real world (i.e. `sane`) RTFs are appropriately smooth allowing us to closely approximate the function with a truncated Taylor series in the proximity of the mean A=.mu., giving ##EQU13##

Substituting this into the integral in (19) and recalling the definitions for mean and variance [6]

allows us to write the approximated expectation as ##EQU14## or more explicitly ##EQU15##

Equation (22) is central to the further development here and, as seen below, its maximization forms the core of all RR/CAPM and RDAA portfolio design solutions.

We immediately note that the derivation of (22) placed no special requirements on the form of the portfolio's p.d.f. It is now the maximization of E(PP.vertline..function.) that drives the selection of .function. no matter how we express the details of .mu..sub.A and .sigma..sub.A. (Several such important alternative expressions are developed below for the more powerful RDAA method that are equally applicable here.)

The solution .function.* to the RR/CAPM portfolio design problem is then stated for .function.= [.function..sub.R *, .function..sub.RF .vertline..function..sub.B] as ##EQU16## and solved subject to the above stated inputs and constraints using one of the many commonly known non-linear programming methods available [17],[7],[8]. For portfolios where E(PP.vertline..function.) is convex in .function. this type of solution unambiguously prescribes the cash-quantified optimal portfolio that correctly reflects the investor's subjective feelings toward assuming monetary risk as defined by his RTF. For non-convex portfolios the solution to (23) may be achieved through one of a number of evolutionary algorithms such as those of the genetic variety [20]. Unfortunately the resulting solution then is merely satisficing and has no guarantee of global optimality.

5.7 Implementation of Risk Direct Asset Allocation by the RDAA Module

The derivation of the RDAA method involves the direct solution of (23) without the constraint that .function..epsilon.CML. In short we now totally bypass the CAPM and its related concepts of the efficient set and capital market line, and directly solve ##EQU17## with more complex and complete constructions of .function., .mu..sub.A, and .sigma..sub.A. Nevertheless (22) remains seminal to all our efforts and is repeated here. ##EQU18##

Before proceeding it is important to note that (22) does require the predicted distribution h() to have computable first and second moments. This requires special care in the use of the more exotic predictive schemas such as offered by the recent Fractal Market Hypothesis and the Coherent Market Hypothesis [16] which in certain "investor sentiment" domains appeal to infinite variance distributions such as the Pareto-Levy to predict security and market returns. Ultimately the validity of (22) holds; for if the investor cannot select a future with an appropriately finite variance (whether it be analytically computed or supplied as a believable heuristic), then, perhaps, no such risky investment should be made. In practice, however, it all comes down to acting on one's belief, and few investors truly believe that even `infinite variance investments` will actually perform with unacceptable likelihoods of disaster awaiting them.

5.7.1 The Simplest Case--Straight Investment of A.sub.1T

The investment fraction vector is now

.function.=[.function..sub.R.sup.T, .function..sub.RF].sup.T (25)

where the risky fractions ##EQU19## We thus see that RDAA permits `free selection` of the risky fractions and the risk free fraction on a par level of emphasis. If its definition should become useful, one could even conceive of a new efficient set for RDAA in the R-.sigma..sub.A plane which set would be tangent to the R axis at R.sub.RF instead of impaling it along the CML. This solution set would be efficient in the sense of providing optimal portfolio designs as one or more input parameters are varied. So we could have a family of RDAA efficient sets to represent the variation in R.sub.M, a given .beta..sub.i, a constraint on .function..sub.R, etc.

To support the user interface designs discussed above for the investor, we will re-formulate the covariance matrix of the risky securities. Let [.rho.] be the correlation matrix corresponding to covS given in (1) such that each element .rho..sub.ij represents the correlation coefficient between

securities i and j defined as ##EQU20## then

cov S=[.rho.].*.sigma. .sigma..sup.T (27)

where .* is the 'corresponding element' matrix multiplication operator and .sigma. is the N-vector of standard deviations. Fractionally allocated rate covariance is again used to compute the total portfolio rate variance as

.sigma..sub.s.sup.2 (.function.)=.function..sup.T cov S.function.(28)

If we now invest A.sub.1T according to the amount allocation vector x with components x.sub.i =A.sub.1T .function..sub.i, i=1,N, then the cash variance of the resulting portfolio will be

.sigma..sub.A.sup.2 (.function.,A.sub.1T)=x.sup.T cov Sx=A.sub.1T.sup.2 .sigma..sub.s.sup.2 (.function.). (29)

The expected portfolio appreciation will increase the expected assets to ##EQU21## where s.sub.i is the appreciation rate of the ith risky security introduced in .sctn.2.2.1. 1. We emphasize again that s.sub.i may be computed from any analytical model or heuristic which is believed to appropriately predict future performance. The only requirement is that the choice of such predictions also yield an appropriate formulation of an equally believable covS, the covariance matrix defined in (27).

With the definition of the first two moments of the predicted assets distribution we compute the expected value of the risk adjusted PP according to (22) using the formulations of .sigma..sub.A and .mu..sub.A given in (29) and (27) respectively. The optimal portfolio then is obtained from the pair ##EQU22## subject to the investor's input of A.sub.lT and the constraint given with (25). N+1 parameters are thus derived from a search over N-space. The amount optimal portfolio design for the simplest case is then given by

x*=A.sub.1T .function.*. (33)

We point out that (31) could have been equally well expressed in terms of x such that .SIGMA.x.sub.i =A.sub.1T and the optimum solution x* derived directly from (32). (The latter form is used in the MATLAB.RTM. program set forth in microfiche Appendix A.

Finally, it is clear that this case also covers solutions for portfolios containing only risky securities since we may set R.sub.RF to some large negative value thereby guaranteeing that no funds will be lent at this rate.

5.7.2 Investment with Combined Lending and Borrowing

Suppose the investor may also borrow funds additional to A.sub.1T at some rate R.sub.B that may or may not be equal to R.sub.RF. It is, of course, reasonable to assume that R.sub.B =R.sub.RF, although this is not a limitation for the RDAA solution we now develop. Frequently, especially in conjunction with other business transactions, there are times when an investor may have the opportunity to borrow some funds at R.sub.B <R.sub.RF. We further assume that the investor is willing to (or may) borrow an amount A.sub.B =.function..sub.B,L1M A.sub.1T where .function..sub.B,L1M is an additional investor supplied constraint such that 0=.function..sub.B =.function..sub.B,L1M. We express the risky 'lending-borrowing' allocation amount vector as

x.sub.LB =A.sub.1T (1+.function..sub.B).function..sub.R (34)

where .function..sub.R is now the column N-vector of 'risky fractions'. The resulting variance expression is then

.sigma..sub.A.sup.2 (.function.,A.sub.1T)=x.sub.LB.sup.T covS x.sub.LB (35)

with the augmented decision (fraction) vector .function.= [.function..sub.R.sup.T,.function..sub.RF,.function..sub.B] retaining the constraints given in .sctn.5.7.1. With lending and borrowing the mean is now ##EQU23##

These values are used to compute E(PP.vertline..function.,A.sub.1T) in (3 1) and incorporated into the optimum solution (in N+1 space) for the N+2 variables ##EQU24##

The resulting policy calls for a total invested amount of A.sub.1 *=A.sub.1T (1+.function..sub.B *) made up of owned and borrowed funds to be invested in the amounts

x.sub.R *=A.sub.1 *.function..sub.R *, x.sub.RF *=A.sub.1 *.function..sub.RF * (38)

We note that A*.sub.1 is `only partially optimized` since at this point the total invested amount is still influenced largely by the investor's selection of A.sub.1T. This requirement will be removed in the subsequent development.

We point out that the above optimal portfolio solution demonstrates RDAA's inherent ability to solve the concurrent lending and borrowing problem without imposing additional restrictions or constraints. Within such a favorable interest rate environment RDAA could discover an investment policy wherein it may behoove the investor to simultaneously reject both counts of the adage "never a borrower or lender be".

5.7.3 The Optimal Amount to Invest

The preceding RDAA solutions robustly cull the Nmember `short list` when certain issues provide no benefits of diversification. However so far we have been forced to invest the entire amounts specified by A.sub.1T and A.sub.1 no matter what the current risk free return R.sub.RF or the historical performance (e.g. reflected by .alpha., .beta., .sigma., covS) of the N securities. Due to its direct approach to maximizing the investor's utility-mapped PP, RDAA may also be configured to select the amount A.sub.1T to be invested subject to the investor supplied constraint that A.sub.1T .epsilon. [0,.function..sub.1,L1M A.sub.T] where 023 .function..sub.1,L1M .ltoreq.1 is termed the investment fraction of total net assets.

We begin by again augmenting the decision vector of fractions so that now

.function.=[.function..sub.R.sup.T, .function..sub.RF,.function..sub.B,.function..sub.1] (39)

is an N+3 vector requiring a search in N+2 space. The RDAA solution here is essentially a replication of the lending/borrowing model of .sctn.5.7.2 with .function..sub.1 A.sub.T substituted for A.sub.1T. We list the relevant relationships without comment.

x.sub.1 =A.sub.T .function..sub.1 (1+.function..sub.B).function..sub.R (40)

.sigma..sub.A.sup.2 (.function.)=x.sub.1.sup.T covS x.sub.l (41) ##EQU25##

These are now used to compute E(PP.vertline..function.) per (22) for obtaining

.function.*=arg max/.function.E)PP.vertline..function.) (43)

The apparent simplicity of the recurring optimization problem statement in these developments may cause one to overlook the fact that the solution has become more non-linear and complex in the decision variables .function. as we have added degrees of freedom and power to the model.

The amount optimal portfolio is obtained by calculating the optimal total amount to invest from owned and borrowed funds

A.sub.l *=A.sub.T .function..sub.l *(1+.function..sub.B *) (44)

which is allocated according to

x.sub.R *=A.sub.T .function..sub.l *(1+.function..sub.B *).function..sub.R *, x.sub.RF *=A.sub.T .function..sub.l *(1+.function..sub.B *).function..sub.RF * (45)

This aspect of the RDAA is the first time that any asset allocation method can unambiguously advise the investor that the short list she has developed is not worthy of the entire amount she is prepared to invest. If RDAA yields .function..sub.l *<.function..sub.1,L1M, then this message strongly recommends a review and revision of not only the short list contents but, perhaps also, a reconsideration of the underlying methodology used to select the N securities.

Before concluding this subsection we introduce a further capability of both RR/CAPM and RDAA that applies uniformly to all solution forms. This is the ability of the investor to specify enforced diversification and/or minimums for all elements of the decision vector .function. such as may be imposed by prudence, corporate policy, or governmental regulations on, say, a mutual funds manager. The implementation of this feature was shown in the user interface 701 of FIG. 7, with the constraint brackets 707. Such sophisticated solutions are obtained by directly supplying an additional set of constraint vectors .function..sub.L and .function..sub.U for the lower and upper bounds respectively. The required constraints are then the by-element vector inequalities

.function..sub.L .multidot..ltoreq..function..multidot..ltoreq..function..sub.u (46)

with the understanding that the elements of both constraint vectors must also satisfy the intrinsic semantic constraints defined above. The response of these algorithms to the constraints provides an additional set of feedback messages to the investor then consisting of either how the invested amount A.sub.l is reduced and/or which securities are rejected from the portfolio or both.

5.7.4 The `Complete` Solution

We conclude this explication by studying how several important additional investor supplied parameters that characterize `real world` investments would be incorporated. As further expanded in .sctn.5.7.5, the inclusion of these parameters is meant to be `complete` only within the scope of this presentation of RDAA and does not imply that RDAA (and RR/CAPM) capabilities are limited to the present embodiment.

The RDAA (and RR/CAPM) methods and implementations presented here may be augmented with an arbitrary number of additional inputs that adjust and tune the actual portfolio costs to the investor. We complete this development by presenting the RDAA model which incorporates all of the above with the inclusion of the following added costs that are often experienced in practice.

.nu..sub.FL --N-vector of front load rates for each security that diminish the actual amounts invested (the inclusion of a back load is a straightforward extension and is demonstrated here through frac.sub.OUT defined below);

.nu..sub.p --the portfolio advisor or management load rate imposed over the investment horizon;

.function..sub.RP =[frac.sub.IN .function..sub.OUT], a simplified application of .nu..sub.p to approximate the usual quarterly portfolio advisor fees computed by charging the going in fraction, frac.sub.IN, of .nu:.sub.p as front load and the coming out fraction, frac.sub.OUT, of .nu..sub.p as a back load at the end of the investment horizon.

The explicit limit fraction .function..sub.RF =.function..sub.RF,L1M where .function..sub.RF,LIM is used to control the fraction of A.sub.1 that will be invested at the risk free rate R.sub.RF .multidot..function..sub.RF,LIM is the (N+1)th element of .function..sub.U in (46).

The portfolio advisor amount charged going in is

A.sub.P,in =.nu..sub.P .function..sub.RP,1 A.sub.1 =A.sub.T .function..sub.1 (1+.function..sub.B).nu..sub.P .function..sub.RP,1 (47)

and the total front load amount is computed as ##EQU26## which yields the risked invested amounts vector components of x.sub.R as

x.sub.R,i =(A.sub.1 -A.sub.P,in).function..sub.R,i (1-.nu..sub.FL,i)=A.sub.T .function..sub.1 (1+.function..sub.B)(1-.nu..sub.P .function..sub.RP,1).function..sub.R,i (1-.nu..sub.FL,i) i=1,N (49)

along with the risk free amount

x.sub.RF =A.sub.T .function..sub.1 (1+.function..sub.B)(1-.nu..sub.P .function..sub.RP,1).function..sub.RF .multidot. (50)

The expected amount of portfolio appreciation at the end of T.sub.1 is ##EQU27## from which we must deduct the loan interest A.sub.B R.sub.B =A.sub.T .function..sub.1 .function..sub.B R.sub.B and portfolio management fee

A.sub.P,out =(A.sub.1 +A.sub.RP -A.sub.P,in).nu..sub.P .function..sub.RP,2 .multidot. (52)

We can now express the expected total assets at the end of T.sub.1 to be

.mu..sub.A (.function.)=A.sub.T -A.sub.P,in -A.sub.FL +A.sub.RP -A.sub.P,out (53)

where we recall the vector of decision variables to be .function.= [.function..sub.R.sup.T,.function..sub.RF,.function..sub.B,.fun ction..sub.1].sup.T with ##EQU28## Now substituting the above definitions gives the investor's expected total assets at the end of T.sub.1 in terms of the decision variables. ##EQU29##

The portfolio amount covariance is then

.sigma..sub.A.sup.2 (.function.)=x.sub.R.sup.T covS x.sub.R (55)

The solution is again obtained by calculating E(PP.vertline..function.) from (22) and using it in the now familiar constrained non-linear programming problem

.function.*=arg max/.function..sub.L .ltoreq..function..ltoreq..function..sub.u E(PP.vertline..function.) (56)

The final portfolio design amounts can be computed from (47) to (55) by substituting the appropriate elements of the optimum allocation vector .function.*.

5.7.5 Value at Risk and RDAA

We conclude by integrating the concept of Value at Risk (VAR) as a natural inequality constraint to the RDAA (and RR/CAPM) solution. VAR is one of the financial industry's latest attempts to bring together the investor's quantitative aversion to risk and the task of portfolio selection without resorting to utility theory per se. Rather than serve as a substitute for monetary utility, VAR is seen in the sequel as a natural augmentation to the incorporation of monetary utility as presented here. The motivation for this feature is based on the notion or Bernstein's [22] "central idea" of risk as "that variability (which) should be studied in reference to some benchmark or some minimum rate of return that the investor has to exceed."

VAR is formally defined as

the amount of money A.sub.VAR such that a portfolio is expected to lose less than this amount over

the investment horizon with probability P.sub.VAR.

It is clear that the prescription is readily accessible to the non-technical investor. Eliciting this pair of values will constrain the optimal .function.* to generating predicted portfolio p.d.f.s that limit the probability of loss P.sub.L while still maximizing the risk compensated return of (22). P.sub.L is defined with respect to an amount A'.sub.T which may be the net current assets A.sub.T, or A.sub.T appreciated by placing the total invested amount A'.sub.T at the risk free rate. P.sub.L is then the probability that the investor's net total assets at the end of the investment horizon will be less than A'.sub.T. In terms of the portfolio's predicted p.d.f. h(), as defined over the amount axis (cf. 22), P.sub.L is the probability mass `to the left of` the A'.sub.T point on the amount axis.

According to the definition of VAR, the required relationship between P.sub.L and P.sub.VAR is then ##EQU30##

Where erf and erfc are the direct and complimentary error functions. Or, expressed in terms of an inequality constraint in non-linear programming, it becomes ##EQU31##

The primary effect on the RDAA solution of including such a constraint is in its ability to set a maximum amount to be invested in the risky portion of the portfolio. A more pronounced secondary effect may also be observed when the investor presents RDAA with a poor short list and/or is adamant about imposing inappropriately high minimum constraint fractions to risky securities. In this case the VAR constraint appeals to the only remaining `policy levers` available to it, namely the investment and borrowing fractions .function..sub.1 and .function..sub.B, and will cause RDAA to unambiguously recommend a reduced total invested amount that yields the specified VAR.

A last benefit of including the VAR constraint involves expanding the ability of RDAA to tolerate `marginally sane` investor RTFs. This more sophisticated benefit comes into play when the investor--perhaps carelessly--inputs reference gamble points that yield unrealistically low Risk Compensation Coefficient values (i.e. one half times the second partial derivative in (22)) over a part of the amount axis--i.e. the RTF's second derivative approaches zero as the risk premium approaches a linear function of predicted net total assets. In this case the portfolio's volatility, .sigma..sub.A, may be discounted too much by (22) if the optimum lies in this region of predicted net total assets. Including the VAR constraint, however, causes RDAA to never overlook or discount this fundamental and overriding measure of risk.

5.8 Summary

In summary, the present invention provides an analytically correct system and method of determining an optimal allocation of an investor's investment assets among any set of investment securities.

It should be clear from the above development that this presentation has not exhausted the complexity of the financial models which may be embedded in the overall RDAA framework. Some obvious and straightforward extensions include marginally taxed dividend yields for certain securities and the inclusion of the investor's current portfolio as part of the N security short list presented to RDAA. With the latter would be included the appropriate capital gains and marginal tax rates along with the costs and current prices of the held securities.

It is also possible to handle more complex formulations of the resulting portfolio's p.d.f. via a direct extension of the risk compensated utility mapping of (22) such as may be required for portfolios that include options and/or derivatives. A more conventional case that may occur in practice for risk tolerant investors is when high variance financial instruments are included which yield a portfolio variance containing a significant probability mass below the `total loss amount` or residual net assets or ##EQU32## In such cases the probability of loss would be over stated in the situations where the maximum loss is restricted to the total amount of the risky investments. This is because the portfolio's (usually gaussian) p.d.f would then be truncated at the total loss amount. In addition, such a truncated p.d.f. has a higher mean and a reduced variance. The necessary modifications to .mu..sub.A and .sigma..sub.A.sup.2 are as follows:

We now derive the portfolio mean and variance expressions corrected for the case when the maximum loss possible is limited to the sum of all invested risky amounts. The situation is depicted in FIG. 12. The original gaussian density for the portfolio is given by ##EQU33##

Where .mu. is the expected value of investor net assets and .sigma. is its standard deviation. Suppose the maximum amount at risk that can be lost is x.sub.R, then the original p.d.f is truncated at x.sub.R as shown in the figure. The truncated probability mass is redistributed to yield a new p.d.f. given by ##EQU34##

The integral in (60) can be expressed in terms of the error function as ##EQU35##

Using this with t=(x-.mu.)/.sigma. in (62) gives us the piecewise defined truncated density as ##EQU36## and its cumulative distribution ##EQU37##

As indicated in Meyer [20], such a truncated gaussian has its mean and variance given by ##EQU38## where the nomalized deviate z.sub.R =(x.sub.R -.mu.)/.sigma. with their respective density and cumulative counterparts ##EQU39##

Expressing the ratio in (64) and (65) as ##EQU40## or in terms of the original portfolio mean variance ##EQU41## lets us write the desired corrected mean and variance for the truncated distribution as ##EQU42## which can be used to re-enter (22) in obtaining the RDAA optimum presented above.

An additional output of interest to the investor is the probability of loss P.sub.L as illustrated in FIG. 12. This same measure of the recommended portfolio is also useful for calculating certain `value at risk` problem formulations (cf. .sctn.5.7.4) and is obtained directly from (63) as ##EQU43##

Finally, since RDAA yields the portfolio's p.d.f. mean and variance in monetary terms, it is possible to compute the total probability P.sub.L of losing money from the recommended investments. The accuracy of this estimated probability is based on the extent to which the portfolio's amount p.d.f. tends to the normal distribution [16]. Within this notion it is possible to conceive of extended RDAA solutions in which the investor adds further constraints to limit this loss probability to a specified level, for example using Value at Risk metrics. In sum, the fimdamental RDAA structure welcomes these kinds of embellishments and extensions.

REFERENCES

[1] Keeney, R. L., Raiffa, H., Decisions with Multiple Objectives: Preferences and Value Tradeoffs, John Wiley & Sons, 1976.

[2] Pearl, J., Probabilistic Reasoning in Intelligent Systems: Networks of Plausible Inference, Morgan Kaufman, 1988, Chpt. 6.

[3] Behn, R. D., Vaupel, J. W., Quick Analysis for Busy Decision Makers, Basic Books, 1982, Chpt. 8.

[4] Ross, S. A., Westerfield, R. W., Jaffe, J. F., Corporate Finance, Irwin, 1993, Chpts 9, 10.

[5] Zwillinger, D., ed., "Standard Mathematical Tables and Formulae--30th Edition", CRC Press, 1996.

[6] Papoulis, A., Probability, Random Variables, and Stochastic Processes, McGraw-Hill, 1965.

[7] Pierre, D. A., Lowe, M. J., Mathematical Programming Via Augmented Lagrangians, Addison-Wesley, 1975.

[8] Grace, A., Optimization Toolbox (MATLAB.RTM.), The MathWorks, Inc., 1992.

[9] Muth, J., "Rational Expectations and the Theory of Price Movements", Econometrica--29, 1961.

[10] Casti, John L., Searching for Certainty--What Scientists Can Know About the Future, William Morrow & Company, 1990.

[11] Sharpe, W., "Capital Assets Prices: A Theory of Market Equilibrium Under Conditions of Risk", Journal of Finance--19, 1964.

[12] Markowitz, Harry, Portfolio Selection, John Wiley & Sons, 1959.

[13] Ross, Stephen A., "The Arbitrage Theory of Capital Asset Pricing", Journal of Economic Theory, December 1976.

[14] O'Shaughnessy, James P., Invest Like the Best, McGraw-Hill, 1994.

[15] Statman, Meir, "How Many Stocks Make a Diversified Portfolio?", Journal of Financial and Quantitative Analysis, September 1987.

[16] Peters, Edgar E., Chaos and Order in the Capital Markets, John Wiley & Sons, 1991.

[17] Press, W. H., et.al., Numerical Recipes in C: The Art of Scientific Computing, Cambridge University Press, 1988.

[18] Sharpe, W. F., "Portfolio Choice" (ver 4October 1996) Copyright .COPYRGT.1995, William F. Sharpe (wfsharpe@leland.stanford.edu).

[19] Meyer, S. L., Data Analysis for Scientists and Engineers, John Wiley & Sons, 1975.

[20] Goldberg, David E., Genetic Algorithms in Search, Optimization, & Machine Learning, Addison-Wesley, 1989.

[21] Bernstein, Peter L., Against the Gods: The Remarkable Story of Risk, Wiley & Sons, 1996.

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United States Patent ***5,784,696***
Melnikoff **July 21, 1998**

Methods and apparatus for evaluating portfolios based on investment risk

Abstract

A portfolio selector for selecting an investment portfolio from a library of assets based on investment risk and risk-adjusted return is provided. The selector chooses a tentative portfolio from the library and determines a risk-adjusted return for the portfolio. The risk-adjusted return is computed by subtracting the average of multiple segment shortfalls from the average of multiple segment performances, over the same segments, based on analysis of market value data for the assets in the portfolio and for a baseline asset. The asset selection and computation is repeated until the risk-adjusted return of the portfolio satisfies criteria derived from preference data specific to an investor. A data storage medium encoded with instructions for performing the method is also provided.

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References Cited [Referenced By]

U.S. Patent Documents

Other References

W.F. Sharpe, Fianancial Analysts Journal, pp. 10-11, Abstract No. D85002332 (INSPEC.), May-Jun. 1985.

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Parent Case Text

CROSS REFERENCE TO RELATED APPLICATION

This application is a continuation-in-part of copending U.S. patent application Ser. No. 08/393,910, filed Feb. 24, 1995.

Claims

1. A machine-readable data storage medium encoded with a set of machine-executable instructions for using a data processing system to perform a method for determining an average relative performance of a first asset relative to a second asset, each of said assets having market value data observable on a periodic basis, said average relative performance based on first value data for said first asset and second value data for said second asset, said method comprising the steps of:

(a) providing said first value data comprising a first plurality of pairs of observations of market value of said first asset, the observations in each pair of observations being spaced apart by a time duration common to all said pairs of observations;

(b) providing said second value data comprising a corresponding second plurality of pairs of observations of market value of said second asset, the observations in each pair of observations being spaced apart by said time duration;

(c) determining, for each pair in said first plurality and a corresponding pair in said second plurality:

(i) a performance of said first asset over said time duration;

(ii) an extent to which said first asset underperforms said second asset over said time duration, which extent when negative is replaced by zero, to yield a nonnegative shortfall;

(d) computing a first average of said performances;

(e) computing a second average of said nonnegative shortfalls;

(f) computing said average relative performance by computing a difference between said first average and said second average; and

(g) communicating to said investor a report comprising information sufficient to identify said first asset and a recommendation whether to execute one or more trades of said first asset, with said recommendation being based on said average relative performance.

2. The machine-readable data storage medium of claim 1 wherein, as encoded, said difference between said first average and said second average is weighted.

3. The machine-readable data storage medium of claim 2 wherein, as encoded, said difference is weighted by a loss-to-gain aversion weight that is specific to an investor.

4. The machine-readable data storage medium of claim 3 wherein, as encoded, said aversion weight varies with the magnitudes of said shortfalls.

5. A machine-readable data storage medium encoded with a set of machine-executable instructions for using a data processing system to perform a method for determining an average relative performance of a first asset relative to a second asset, each of said assets having market value data observable on a periodic basis, said average relative performance based on first value data for said first asset and second value data for said second asset, said method comprising the steps of:

(a) identifying pairs of first value data and pairs of second value data, each said pair of first value data corresponding in time with one of said pairs of second value data, each pair comprising a first

datapoint and a second datapoint;

(b) for each pair of first value data and each corresponding pair of second value data:

(i) computing a nonnegative extent to which said first asset underperforms said second asset; and

(ii) computing a performance of said first asset equal to a percentage increase in asset value from said first datapoint to said second datapoint;

(c) determining said average relative performance by computing a function of said nonnegative extents and said performances; and

(d) communicating to said investor a report comprising information sufficient to identify said first asset and a recommendation whether to execute one or more trades of said first asset, with said recommendation being based on said average relative performance.

6. The machine-readable data storage medium of claim 5 wherein, as encoded, said datapoints in each of said pairs of value data represent value data that are spaced apart in time by a time duration that is the same for all of said value data pairs.

7. The machine-readable data storage medium of claim 6 wherein, as encoded, computing a function of said nonnegative extents and said performances includes performing a weighting by a loss-to-gain aversion weight that is specific to an investor.

8. The machine-readable data storage medium of claim 7 wherein, as encoded, said aversion weight varies with the magnitudes of said nonnegative extents.

9. The machine-readable data storage medium of claim 6, further comprising encoded instructions for making an investment recommendation based on said average relative performance.

10. The machine-readable data storage medium of claim 6, further comprising encoded instructions for executing an asset trade in a market in which said first asset is traded, said trade based on said average relative performance.

11. A machine-readable data storage medium encoded with a set of machine-executable instructions for using a data processing system to perform a method for determining an average relative performance of a first asset relative to a second asset, each of said assets having market value data observable on a periodic basis, said average relative performance based on first value data for said first asset and second value data for said second asset, said method comprising the steps of:

(a) identifying pairs of first value data and pairs of second value data, each said pair of first value data corresponding in time with one of said pairs of second value data, each pair comprising a first datapoint and a second datapoint, said datapoints in each of said pairs of value data representing value data that are spaced apart in time by a time duration that is the same for all of said value data pairs;

(b) for each pair of first value data and each corresponding pair of second value data:

(i) computing a nonnegative extent to which said first asset underperforms said second asset; and

(ii) computing a performance of said first asset equal to a percentage increase in asset value from said first datapoint to said second datapoint;

(c) determining said average relative performance by computing a function of said nonnegative extents, of said performances, and of a loss-to-gain aversion weight that is specific to an investor; and

(d) communicating to said investor a report comprising information sufficient to identify said first asset and a recommendation whether to execute one or more trades of said first asset, with said recommendation being based on said average relative performance.

12. A machine-readable data storage medium encoded with a set of machine-executable instructions for using a data processing system to perform a method for determining shortfall in performance of a first asset relative to a second asset, each of said assets having market value data observable on a periodic basis, said shortfall based on first value data for said first asset and second value data for said second asset, said method comprising the steps of:

(a) identifying pairs of first value data and corresponding pairs of second value data, each pair spanning a time period, said time period spanned by each pair of first value data overlapping with said time period spanned by at least one other pair of first value data, and said time period spanned by each pair of second value data overlapping with said time period spanned by at least one other pair of second value data, each pair comprising a first datapoint whose value represents the market value of said first or second asset on an earlier date and a second datapoint whose value represents the market value of said first or second asset on a later date;

(b) for each pair of first value data and each pair of corresponding second value data, computing a nonnegative extent to which said first asset underperforms said second asset, to yield a plurality of nonnegative extents;

(c) averaging said nonnegative extents to yield said shortfall; and

(d) communicating to said investor a report comprising information sufficient to identify said first asset and a recommendation whether to execute one or more trades of said first asset, with said recommendation being based on said shortfall.

13. The machine-readable data storage medium of claim 12 wherein, as encoded, said datapoints in each of said pairs of value data represent value data that are spaced apart in time by a time duration that is the same for all of said value data pairs.

14. The machine-readable data storage medium of claim 1, claim 5, claim 11 or claim 12, said data storage medium being magnetic.

15. The magnetic machine-readable data storage medium of claim 14, said data storage medium being a floppy diskette.

16. The magnetic machine-readable data storage medium of claim 14, said data storage medium being a hard disk.

17. The machine-readable data storage medium of claim 1, claim 5, claim 11 or claim 12, said data storage medium being optically readable.

18. The optically readable storage medium of claim 17, said data storage medium being a CD-ROM.

19. The optically readable data storage medium of claim 17, said data storage medium being a magneto-optical disk.

Description

BACKGROUND OF THE INVENTION

This invention relates to methods and apparatus for evaluating portfolios based on the risk and risk-adjusted return of investments.

Traditionally, investing has been difficult for the typical individual investor, particularly when the investor wishes to invest in a number of different investments for purposes of diversification but has a limited amount of funds to invest. The problem is exacerbated by the fact that most individual investors have neither the understanding nor the resources to properly measure the risk and return of investments.

Considering investment in stocks as illustrative of the general problem posed above, the advent of stock mutual funds in recent years has made it substantially easier for the individual investor to achieve the goal of diversification on a limited budget. The fact that a fund manager assumes the responsibility, which would otherwise be the investor's, of researching and trading the stocks of individual companies has contributed significantly to the widespread popularity of mutual funds as a convenient vehicle for investing in the stock market.

Nonetheless, since the nature of mutual funds varies from fund to fund, the individual investor still faces the problem of evaluating and comparing the various funds available for investment, particularly from the standpoint of return and risk. For example, one fund may be managed more aggressively than other funds, and for this reason might be considered more (or less) desirable for the investor. Accordingly, a fully understandable method for appropriately measuring the returns and risks of individual mutual funds, and thereby aiding in their selection, is needed by individual investors.

It is desirable, at the outset, to clarify the meaning of two key terms which are important to any understanding of this subject--"risk" and "volatility". They are often used interchangeably, even though they really represent very different basic concepts:

The term "risk" in general common usage means the possibility of an undesired event, generally as a result of random forces. It represents the taking of adverse chances; and implies a threat of loss of one's life, health, property or the like.

Investment risk and investment volatility are two different concepts which should be distinguished:

Investment risk is strictly downside, relating to the possibility of loss, underperformance, or shortfall, as defined in each circumstance.

Investment volatility is variability, both upside and downside. Although there may be some relationship between these two different concepts, it cannot be a simple one, as illustrated by the following example of two hypothetical investment funds:

Monthly Rate of Total Return - %

Month	1	2	3	4	5	6	Etc.
Fund A	1.0	1.5	0.5	2.0	0.0	3.0	. . .
Fund B	0.5	0.75	0.0	0.5	0.0	0.75	. . .

In each month, the Fund A return is at least equal to the Fund B return, and in most months it is substantially higher than the Fund B return; but in no sense does Fund A indicate a greater risk than Fund B, even though it is clearly more volatile than Fund B: Fund A additional volatility is all on the upside, which poses no risk.

For individual investors, as well as for institutional investors, investment risk is a measure of the likelihood and the extent of underperforming a preestablished standard, or target rate of total return. Mathematically, that can be represented by the expectation of underperformance (the sum of products of (a) the likelihood of each event of underperformance and (b) the extent of underperformance in

each event, a concept very similar to the much better known expression "expectation of life").

Essentially two methods of measuring investment risk are in current use.

(1) The Modern Portfolio Theory (MPT) Method. It is a cornerstone of Modern Portfolio Theory, which is based on the pioneering work of Harry Markowitz (in 1952) and William F. Sharpe (in 1963), who shared the 1990 Nobel Prize in Economics for that work. The variability of monthly returns over a short period, like 3-5 years, is indicated by the statistical measure known as the standard deviation (the square root of the mean of the squares of the differences between each monthly return and the mean of all monthly returns in the period). Under this method, the riskiness of a fund is indicated by dividing (1) the standard deviation so determined for the fund, by (2) the comparable standard deviation for a market index (such as the S&P 500, for Stock Funds, and the Lehman Aggregate Bond Index, for Bond Funds, or sometimes an index composed 50% of each of these 2 indexes, to provide wider comparability). The higher the result (which is sometimes referred to as .beta.), the greater the "risk". The shortcomings of this approach are fundamental, for it:

(a) Measures only the variability of very short-term (monthly) returns, or volatility.

(b) Recognizes variability in returns in both directions, up and down.

(c) Compares the short-term variability in returns of a fund with that of the corresponding stock market (or bond market or both).

(d) Does not recognize the potential impact of sales loads.

(e) Does not provide any way to recognize the potential impact on risk of taxes on the investor.

(f) Does not provide any way of comparing funds in different asset classes (e.g. stocks vs bonds).

(g) Does not measure investment loss at all, which we believe is the essential meaning of investment risk.

(h) Does not correspond to any other meaning of "risk" in the real world.

(i) Does not lend itself to development of an understandable measure of risk-adjusted return.

(j) Does not permit an investor to apply his personal loss-to-gain aversion weight (explained below).

(k) Is not understandable by individual investors.

(2) The "Morningstar" Method. This was developed in about 1985 by Morningstar Inc., a Chicago financial publishing service, and is the other method of assessing risk in current use. In the original version of the method, a fund's risk-adjusted return is determined as follows:

First, the fund's total return, or "performance" over a specified analysis period is calculated, by determining the percentage increase in value over that period of an investment in the fund, including the result of prompt reinvestment in the fund of any distributions made available by the fund in the period. If, for example, the analysis period is five years, from Oct. 31, 1989 to Oct. 31, 1994, the cumulative performance is computed by subtracting the value of the investment on Oct. 31, 1989 from its value on Oct. 31, 1994, and then dividing the result by the value of the investment on Oct. 31, 1989. The resulting number, i.e., the cumulative change in value, or "performance", is first annualized, and is then divided by the average of the corresponding annual rate of performance for the same period of all the funds included in the broad class of funds to which it belongs (e.g., stock funds, taxable bond funds, municipal bond funds, or hybrid funds), to yield a relative return statistic for the given 5-year period.

Next, the fund's risk, relative to a specified standard, or predetermined baseline rate of return, is

calculated. The Morningstar method uses the three-month Treasury Bill ("T-Bill") rate as its standard baseline or "target" rate of return. Continuing with the example of a five year analysis period, two quantities are computed for each of the sixty calendar months that comprise the five year period: (1) the monthly percentage increase in the value of an investment in T-Bills (T-Bill monthly performance), and (2) the monthly percentage change in the value of an investment in the fund (fund monthly performance). Specifically, in the example indicated above, the performance of the T-Bills and the fund are calculated for November 1989, December 1989 and so on, up to October 1994, to yield two sets of sixty monthly performance numbers. Then, for each of the sixty months, the fund performance is subtracted from the T-Bill performance to yield the extent to which the fund underperformed the T-Bill (the "shortfall") during that month. If the resulting difference is negative, it is set to zero, so that, in effect it is assumed that the fund performance was equal to the T-Bill performance in such months, and no recognition is given to any excess of the fund performance over the T-Bill performance in such months. The resulting sum of sixty numbers, consisting of shortfalls and zeros, is divided by 60, to yield the average monthly shortfall, which is the risk statistic for the fund. This risk statistic is then divided by the average corresponding risk statistic for the broad class of funds to which it belongs (mentioned above) to yield the relative risk statistic for the fund.

The fund's relative risk-adjusted return for the analysis period is then determined by subtracting the fund's relative risk statistic from the fund's relative return statistic.

The Morningstar method properly undertakes to measure investment risk in terms of shortfall rather than merely any variability. However it suffers from a number of fundamental shortcomings.

First, measuring shortfall on a month-by month basis, but assuming a fund return equal to the T-Bill return for all months in which the fund did not underperform T-Bills, in effect bases its shortfall or risk calculation on the assumption that what is being analyzed are the results of 60 investments in the fund as well as in T-Bills, each of which is held for only one month. This is evident from the following example which emphasizes the importance, in measuring investment risk, of the concept of the investment holding period. The following Table shows the monthly rate of excess return (i.e. over the T-Bill rate) for 2 hypothetical funds:

Monthly Rate of Excess Return - %

Month	1	2	3	4	5	6	Etc.
Fund C	2.0	-0.2	3.0	-0.4	4.0	-0.6	. . .
Fund D	1.0	-0.1	1.5	-0.2	2.0	-0.3	. . .

In each month, Fund C has double the Fund D rate of excess return. Measured by these rates, Fund C has twice the volatility of Fund D. But Fund C is riskier than Fund D only for an investment made for an even-numbered month and held for only that month. For any other investment, Fund C clearly indicates less risk as well as higher returns than Fund D. But the Morningstar method would consider Fund C as riskier than Fund D, as it would recognize the shortfalls in the even-numbered months, and, in effect, ignore the excess returns in the odd-numbered months. The ambiguity and confusion in the meaning and significance of the Morningstar method is evident in the following sentence, which is part of the explanation of the method that appears in the New York Times every Saturday: "The risk rating reflects downside volatility which is measured during months in which a Fund underperformed Treasury Bills." The one month investment duration hardly reflects the typical investment. By basing the calculation on multiple one-month periods, it fails completely to recognize the decreasing risk related to longer investment holding periods.

Second, the Morningstar return statistic is calculated on a different assumption than is used for the risk statistic. Although risk is calculated by averaging shortfall results based on 60 investments each held for only a single month, the return statistic is calculated on the assumption of a single

investment held for a 60-month period. Subtracting two quantities determined on different assumptions as to the investment holding period is inconsistent and results in an intuitively unsatisfying measure for risk-adjusted return.

Third, the return statistic is measured on a basis that recognizes sales loads, but the risk statistic does not recognize sales loads--another inconsistency.

Fourth, the two statistics, for risk and return, are expressed in different "units" or "currencies" ("relative risk" and "relative return"). So the result of the subtraction is a "hash" quantity that does not correspond to any recognizable concept in the real world. This numerical result is not even disclosed; instead icons in the form of stars are used to characterize the "star rating" of a fund (from one to five) to indicate the "class" of risk-adjusted return to which the fund belongs.

Fifth, the return statistic is based on a single observation, of the entire analysis period (in the example above, from Oct. 31, 1989 to Oct. 31, 1994); it is therefore inherently unstable.

Sixth, effective at the end of April 1994, the Morningstar method was revised, to change the return statistic. Originally it was the ratio of the fund annual return for the entire analysis period (e.g. 5 years) to the corresponding average of the annual returns for the period of all funds in its class (all stock funds, etc.). In the revision, the numerator of the ratio became the excess of the fund return over the T-Bill return for the period; the denominator similarly became the excess of the average return of all funds in its class over the T-Bill return, but with a proviso: the resulting denominator cannot be less than the T-Bill return for the period. This is a further degree of abstraction, making understanding by the individual investor less possible.

Seventh, the Morningstar procedure, in some circumstances, obscures the possibility that a fund's highly superior return may more than offset its somewhat higher risk.

Eighth, this procedure does not lend itself to a comparable, sound method of developing risk-adjusted returns on a basis recognizing taxes on the investor.

Ninth, many investors wish to select a portfolio of mutual funds of different classes, including funds invested in stocks, taxable bonds, and municipal bonds, as well as combinations of all three. More specifically, investors often want to select funds which tend to "hedge" one another, i.e. that offset one another in such a way as to reduce the aggregate investment risk. For example, if one mutual fund has historically yielded somewhat higher returns during periods when another fund has performed poorly, their combination in such periods would have resulted in a hedged portfolio of funds with reduced risk. The Morningstar method does not permit the analysis of return, risk and risk-adjusted return for a portfolio comprising more than one class of mutual fund.

SUMMARY OF THE INVENTION

In view of the foregoing, it is an object of this invention to provide methods and apparatus for defining, measuring, and expressing investment results, including risk, that are fully and readily understandable by the individual investor and that provide a meaningful measure of risk-adjusted return which can be expressed in an understandable manner.

More specifically, it is an object of this invention to provide methods and apparatus that are based on a definition of risk as being the likelihood and extent of underperforming a preestablished standard of return; that recognize the significance, for investment risk, of the length of the investment holding period; and that measure returns and risks on a consistent basis.

It is another object of this invention to provide methods and apparatus that are universally applicable, to all asset classes and funds, on a uniform basis.

It is also an object of this invention to provide methods and apparatus that recognize the impact of sales loads, where applicable, on both returns and risks, in a consistent manner. This includes

methods and apparatus to evaluate the impact on risks and risk-adjusted returns of different schedules of sales loads available for the same net investment results (the so-called "A", "B", "C", and "D" mutual funds).

It is a further object of this invention to provide methods and apparatus that recognize the impact of taxes payable by the investor on both returns and risks, in a consistent manner (required for sound evaluation of portfolios that include municipal bond funds and other mutual funds).

It is still another object of this invention to provide methods and apparatus that permit an individual investor to apply easily a personal loss-to-gain aversion weight in calculating risk-adjusted return.

It is a further object of this invention to provide methods and apparatus that accommodate custom-tailored analyses for individuals prescribing alternative target rates of return or variable loss-to-gain aversion weights that depend on the degree of the shortfall.

It is a still further object of this invention to provide methods and apparatus that assist in the selection of a portfolio of mutual funds from a given library of candidate funds, by optimizing the degree of indicated fidelity of the chosen funds to the investor's expressed preferences in terms of risk and risk-adjusted return, exhibited for a selected investment holding period in a given period of analysis.

These and other objects of the invention are accomplished by providing methods and apparatus for selecting from a library of assets an investment portfolio of one or more assets, the selection being based on asset value data for the assets, target value data for a target asset, and investor preference data reflecting the preferences of the investor or group of investors for whom the portfolio is being selected. The asset value data and the target value data provided to the system take the form of pairs of datapoints, with the datapoints in each pair being spaced apart by a time duration common to each of the pairs.

An asset, or a set of assets and their relative proportions, is selected from the library of assets to form a tentative investment portfolio. The average relative performance of the tentative portfolio is computed by first forming a weighted sum of the asset value data for the assets forming the tentative portfolio, resulting in portfolio value data in the form of pairs of datapoints. The weights used to form the sum correspond to the relative proportions of the assets in the tentative portfolio. Second, for each pair of portfolio data and target data, a performance of the tentative portfolio over the common time duration, as well as a nonnegative shortfall (defined to be the extent to which the tentative portfolio underperforms the target asset over the common time duration), are calculated. The performances are averaged, as are the shortfalls. Subtracting the latter from the former yields the average risk-adjusted return of the tentative portfolio (assuming a loss-to-gain aversion weight of 1.0).

The performance of the tentative portfolio is compared to criteria derived from the investor preference data. If the criteria are satisfied, the tentative portfolio is designated as the investment portfolio. If not, a new tentative portfolio is selected and the processing sequence is repeated in an iterative manner until the criteria derived from the investor preference data are satisfied. The designated investment portfolio is then communicated to the investor.

Also provided is a machine-readable data storage medium on which is encoded a set of machine-executable instructions for performing the method.

BRIEF DESCRIPTION OF THE DRAWINGS

The above and other objects and advantages of the invention will be apparent upon consideration of the following detailed description, taken in conjunction with the accompanying drawings, in which like reference characters refer to like parts throughout, and in which:

FIG. 1 is a schematic view of a first preferred embodiment of a hardware system according to the present invention;

FIG. 2 is a table showing one form that mutual fund data may take;

FIG. 3 is a table showing one form that target rate data may take;

FIG. 4 is a table showing one form that additional "static" information, including sales loads, if any, as well as investor preference indications may take;

FIGS. 5A, 5B, and 5C (collectively, "FIG. 5") are a flow diagram of the core processing method of the system;

FIGS. 6A, 6B, 6C, 6D, 6E, 6F, and 6G (collectively, "FIG. 6") are a representative printout of fund values, target rate values, and intermediate data from the computation of risk, return, and risk-adjusted return, calculated for a one-month holding period;

FIGS. 7A, 7B, 7C, 7D, 7E, 7F, and 7G (collectively, "FIG. 7") are a representative printout of fund values, target rate values, and intermediate data from the computation of risk, return, and risk-adjusted return, calculated for a one-year holding period;

FIGS. 8A, 8B, 8C, 8D, 8E, 8F, and 8G (collectively, "FIG. 8") are a representative printout of fund values, target rate values, and intermediate data from the computation of risk, return, and risk-adjusted return, calculated for a three-year holding period;

FIGS. 9A, 9B, and 9C (collectively, "FIG. 9") are a Summary of the results of the application of the system to 3 mutual funds, in each case for 3 holding periods: one month, one year and three years;

FIG. 10 is a schematic view of a second preferred embodiment of a system according to the present invention;

FIG. 11 is a cross-sectional view of a magnetic data storage medium encoded with a set of machine-executable instructions for performing the method according to the present invention; and

FIG. 12 is a cross-sectional view of an optically readable data storage medium encoded with a set of machine-executable instructions for performing the method according to the present invention.

DETAILED DESCRIPTION OF THE INVENTION

The methods and apparatus of the invention can be readily applied to all types of investment for which market values and liquidation facilities are available on a regular, periodic basis, including individual securities, mutual funds of all kinds, as well as closed-end funds, variable annuities, commodity funds, separate accounts, commingled funds, and any portfolio consisting of a combination of these. To simplify the terminology in the following discussion, it is assumed that all investments are mutual funds.

The invention provides that the term "risk" means the likelihood and the extent of shortfall, i.e. of underperforming a preestablished standard level of return, which may be called the baseline asset or target rate of return. In general, there is broad flexibility in setting the standard level, which may be set on any basis by the investor. A generally useful target rate for most investors is the rate of a completely risk-free investment in the currency of the investor, which, for a U.S. investor, would be the contemporaneous rate on 3 month U.S. Treasury Bills ("T-Bills"). Accordingly, the T-Bill rate is used for illustration in the description that follows. (However, for a less risk-averse investor who may be willing to tolerate greater risk in seeking larger returns, the target rate may be a zero percent return or even a negative return. Alternatively, for more risk-averse investors, the target rate may be a fixed or variable rate substantially higher than the T-Bill rate.)

To measure risk on this basis, it is particularly preferable to follow a probabilistic approach, using rolling periods of uniform length, representing the assumed duration of the investment holding period, which should reflect durations representative of typical investor behavior. Shown as examples

only in the description below are results for assumed investment holding periods of 1 month, 1 year, and 3 years (other periods should also be considered but are not shown).

To obtain a measure of returns to which the measure of risk can be related, the returns of an investment are measured on substantially exactly the same probabilistic basis used for measuring risks or shortfall.

In the description which follows, the basic annual risk-adjusted rate of return is defined as the average annual unadjusted rate of return reduced by the average annual rate of shortfall. Accordingly, it is expressed in the same "currency" (i.e. annual rate of return) as the unadjusted annual rate of return. This basic measure incorporates the proposed standard loss-to-gain aversion weight of 1.0, which means that it accords with the findings of behavioral economists that, in general, to individuals, a potential loss of $1 fully offsets an equally likely potential gain of $2, which is the basis used in the material which follows. Other loss-to-gain aversion weights may, of course, be used, as discussed below.

As indicated above, this method is completely self-contained, and does not require any reference to external data, such as results for market indices.

In accordance with the invention, substantially the exact same procedure is followed for all kinds of mutual funds (as well as other investments for which market values and liquidation facilities are available on a regular periodic basis) making it universally applicable across different asset classes, and thus suitable for risk analysis of portfolios including different asset classes.

The invention can be readily applied to recognize the impact of sales loads, in a consistent manner, on both returns and risks, as well as on risk-adjusted returns. In the detailed analysis of a single mutual fund which is exhibited in FIGS. 6-8 (discussed in more detail below), the impact of sales load is recognized. However, even for mutual funds with a sales load, an analysis which does not recognize sales loads conveys useful information to some investors, such as an investor who may be exempt from the sales load (e.g., an employee of the fund manager), or an investor who has already absorbed the sales load but is uncertain whether to retain the existing investment in the fund. Accordingly in FIGS. 6-8, as well as in FIG. 9, which summarizes the results of analysis for three mutual funds, results are shown both without and with recognition of sales loads, thereby illustrating the significance of recognizing sales loads for the purpose of measuring returns, risks, and risk-adjusted returns.

In recent years, some mutual fund organizations have made available so-called "A", "B", "C", and "D" funds, for which the same underlying assets are made available with different sales loads, including front-end, back-end, and 12b-fees. The invention can be readily applied to compare the risks and risk-adjusted returns of each of such arrangements.

The invention also can be readily adapted to recognize the impact of federal income taxes payable by the investor, and to express the results in a format which is consistent with the format for results which do not recognize such taxes. Such an adaptation is required for sound evaluation of the risk and risk-adjusted returns of portfolios which include municipal bond funds as well as other mutual funds. (The required adaptation is described below, but not the results.)

The preferred method of the invention is as follows:

It is particularly preferable to make a significant number of observations of the performance of the investment, on a consistent basis, since the only way to measure investment risk is on a probabilistic basis. Accordingly, a fund investment record is studied by dividing the total record of the period studied into "rolling periods", or segments (which are generally overlapping), each of a length in time equal to the fixed investment holding period being examined (e.g., 1 month, 1 year, or 3 years) and each ending with a month in the analysis period.

The segmenting is achieved by identifying pairs of fund value observations (i.e., pairs of

"datapoints") and pairs of target value observations, or datapoints, that correspond to pairs of calendar dates (when the stock exchanges were open) spaced one holding period apart, i.e., spanning the segment. For each segment, or equivalently for each such pair of dates, the performance of the fund over the time spanned is computed by calculating the percentage change from the first fund value datapoint to the second fund value datapoint in the pair. Next, if the holding period is greater than one year, the fund performance is annualized. If the holding period is less than one year, the annualization step is performed later. (If the holding period is exactly one year, then the performance is already annualized.) Finally, the performance is adjusted to account for the fund's sales load, if one exists, with the sales load adjustment percentage being annualized beforehand if the holding period is greater than one year.

A similar computation yields the performances of the target asset over the same segments. Specifically, the performance across the pair of target rate datapoints defining each segment is computed by subtracting and dividing in exactly the same manner as described above for the fund value datapoints, as is the holding-period-dependent annualization step. However, sales loads are not taken into account, since the typical target asset (e.g., T-Bills) does not have a sales load.

The load-adjusted fund performance is subtracted from the target performance (with any negative result being reset to zero), to yield the nonnegative load-adjusted "shortfall", or extent to which the fund was outperformed by the target asset. The load-adjusted fund performances and shortfalls for each of the segments are then averaged, to yield the average load-adjusted return E(1) and the average load-adjusted shortfall E(2). The risk-adjusted return is then computed by performing the following weighted subtraction:

E(1)-W.times.E(2)

where W represents the investor's loss-to-gain aversion weight. (For purposes of the remaining description, it is assumed that W=1.0.) If the holding period is less than one year, the risk-adjusted return so determined is annualized to yield the annual risk-adjusted return of the fund.

The following example of a specific application of the invention, which is helpful in providing a detailed description of the invention, is based on a number of specified conditions. It is evident that many other applications of the invention can be designed, based on other specified conditions, and the procedure modified accordingly within the scope of the invention. The specified conditions of the example that follows are that the analysis should:

1. Be applied to three different mutual funds (chosen were the Mathers Fund, the Fidelity Magellan.RTM. Fund, and the AIM Constellation Fund; the Mathers Fund is no-load; the other two have sales loads. Full details of the application are shown only for the Fidelity Magellan.RTM. Fund in FIGS. 6-8; a Summary of the results for all three funds is shown in FIG. 9);

2. Be applied to each fund on the basis of three different investment holding periods (to indicate the fundamental significance of the investment holding period for the measurement of risk): 1 month, 1 year and 3 years. (The 1 month holding period is analyzed not because it is considered a typical investment holding period, but because the two methods of measuring investment risk which are in current use, as described above, are based--improperly, according to the present invention--on an analysis of monthly returns.);

3. Be applied both (a) without recognizing sales loads, and (b) recognizing sales loads;

4. Be based on 84 observations for each holding period; and

5. Include observations ending with the end of the last business day of each month, beginning with July 1986 and terminating with June 1993.

The preferred system and method of the invention are now described in detail in connection with the accompanying Figures.

The present invention may be implemented on a variety of computer systems--ranging from a modest personal computer (such as those based on the 80.times.86 series of microprocessors originally developed by Intel Corporation, of Santa Clara, Calif.) equipped with a spreadsheet program such as LOTUS.RTM. 1-2-3 (available from Lotus Development Corporation, of Cambridge, Mass.), if the risk-adjusted returns for a small handful of mutual funds are to be determined, to a supercomputer, if an investment portfolio is to be created from a library comprising thousands of funds. (As of 1995, over 7,000 mutual funds were available in the U.S.) An exemplary computer hardware system 10 on which the present invention may be implemented is shown in FIG. 1.

In FIG. 1, which shows a first preferred embodiment of apparatus according to the invention, system 10 includes a computer 11 comprising a central processing unit ("CPU") 20, a working memory 22 which may be, e.g, RAM (random-access memory) or "core" memory, mass storage memory 24 (such as one or more disk drives or CD-ROMs), one or more cathode-ray tube ("CRT") display terminals 26, one or more keyboards 28, one or more input lines 30, and one or more output lines 40, all of which are interconnected by a conventional bidirectional system bus 50.

Input hardware 36, coupled to computer 11 by input lines 30, may be implemented in a variety of ways. Asset value data may be inputted via the use of a modem or modems 32 connected by a telephone line or dedicated data line 34 to an online financial service. Alternatively or additionally, the input hardware 30 may comprise CD-ROMs or disk drives 24. In conjunction with display terminal 26, keyboard 28 may also be used as an input device, in order to input investor preference data, for example.

Output hardware 46, coupled to computer 11 by output lines 40, may similarly be implemented by conventional devices. By way of example, output hardware 46 may include display terminal 26 for displaying the risk-adjusted performance of a fund or the selected investment portfolio. Output hardware might also include a printer 42, so that hard copy output may be produced, or a disk drive 24, to store system output for later use. Where asset trades are to be executed based on an investment recommendation, the trading information may be transmitted over telephone or dedicated data lines 34, possibly with the use of modem 32, to cause the trades to be executed.

In operation, CPU 20 coordinates the use of the various input and output devices 36, 46, coordinates data accesses from mass memory 24 and accesses to and from working memory 22, and determines the sequence of data processing steps. Specific references to components of the hardware system 10 are included as appropriate throughout the following description of the processing steps carried out by the hardware system.

A flow diagram of a routine for computing the return, risk and risk-adjusted return of a mutual fund is provided in FIG. 5, and printouts of the application of the routine to a specific mutual fund history is shown in FIGS. 6-8. A Summary of the results of the application of the system to 3 mutual funds is shown in FIG. 9.

Referring to FIGS. 1 and 5, at step 110, CPU 20 receives as input over bus 50 data 210 corresponding to the candidate mutual funds to be processed. Data 210 is received by bus 50 either from modem 32 connected to an online service, or from storage memory 24. To measure investment performance for a period ending with any month requires a "look-back" procedure, to determine the starting date for the investment holding period. Accordingly, in view of specified conditions 2, 4 and 5, to measure the results of a 3-year holding period ending with July 1986, it is necessary to obtain data going back 3 years earlier, to 1983.

FIG. 2 illustrates one form that the mutual fund data 210 may take. In FIG. 2 columns 220 and 230 respectively represent (1) a series of calendar months 220; and (2) an index of the then current accumulated net asset values ("values") 230 of an investment previously made in a mutual fund, including the result of promptly reinvesting in the fund any distributions made by the fund. As with any series of index values, it is their relative values, and not their absolute values that have meaning. The calendar months in column 220 represent the end of the last day in that month on which the stock

exchanges were open. Calendar months 220 and fund values 230 are also shown, respectively, in the third and sixth columns of FIGS. 6 and 7, and in the third and seventh columns of FIG. 8.

Optionally, the mutual fund data may be augmented to include additional columns of data (such as columns 240 and 250 of FIG. 2) for additional mutual funds.

Referring again to FIGS. 1 and 5, CPU 20 receives target rate value data 310 as input at step 112, again from modem 32 or mass storage memory 24, over bus 50. Target rate value data 310 are periodic data representing the accumulated value of an investment to be used as a preestablished standard, baseline or target of performance, below which fund results will be considered underperformance or shortfall. As indicated above, in this presentation, the target rate accumulated value represents investment in the contemporaneous rate on 3-month Treasury bills ("T-Bills").

FIG. 3 shows the same calendar months 220 as appear in fund data 210, as well as the corresponding accumulated values 330 on such dates of an investment previously made and periodically rolled over in 3-month T-Bills. Target rate values are also shown in the fourth column of FIGS. 6-8.

Both the mutual fund value data 210 and the target rate value data 310 are readily available from a number of public sources, including electronic on-line financial services such as Bloomberg, Reuters and Dow Jones Telerate, as well as in other formats from financial services such as Lipper Analytical Services and Morningstar. Some or all the data 210 and 310 may also be available in CD-ROM format.

Referring back to FIGS. 1 and 5, a variety of other static data 405 are received by CPU 20 at input step 114. As shown in FIG. 4, data 405 include sales load amount 420 for each fund appearing in the fund data 210 (including both front-end sales load and contingent deferred sales load, if any), holding period length 410, loss-to-gain aversion weight 450 (explained below) and any additional risk tolerance data 740. The holding period length 410 is the length of time that a mutual fund investment is assumed to be held, before it is sold. Static data 405 are received by CPU 20 from mass storage memory 24 or keyboard 28, over bus 50.

To ensure proper operation of the system 10, the analysis period should be chosen so that it is long enough, on the one hand, to cover enough "observations" (i.e., an investment purchased on a "starting date", held for a "holding period" and sold on an "ending date") that the system can produce reliable output, but short enough, on the other hand, to avoid basing the output on information that is so old that it may no longer be relevant; and to ensure that more recently introduced funds, with shorter histories, can be processed by the system when they have been operating for a reasonable period.

In view of these considerations, it is suggested that at least 36 observations be made for each holding period, so that the minimum total period for which data would have to be available in mutual fund value data 210 and target rate value data 310 would be 36 months for a one-month holding period, 47 months for a one-year holding period and 71 months for a three-year holding period.

The data received by the system at input steps 110, 112, and 114 are stored by the system at storage register 116 in CPU 20, as shown in FIG. 5.

Continuing with FIG. 5, as the internal operation of the system proceeds, it is helpful to refer, periodically, to the printouts of the system in FIGS. 6-8, as well as to introduce notation which will assist in describing the processing of the basic data which has been stored in storage register 116. Specifically, the observation number of an entry in the printouts will appear as a subscript in the following description, so that, for example, "310.sub.12 " refers to the target rate value 310 that corresponds to the twelfth observation (see column (2)) in the printouts. Likewise, "C.sub.8 " refers to the change percentage "C" (defined below) that corresponds to the eighth observation.

At step 190, in FIG. 5, CPU 20 initializes to zero an index variable "E", representing the observation currently being processed, which always represents the ending month of an investment holding period, and initially corresponds to observation number zero shown in col. 2 of FIGS. 6-8, to item

number 36 shown in col. 1, as well as to June 1986 in col. 3. Index variable E is used primarily as a counter of the observations, which, according to the fourth specified condition, will go as high as 84.

At step 191, CPU 20 increments E by one, to observation 1, which corresponds to item No. 37 and July 1986. At step 118, CPU 20 determines, by a "look-back" procedure, the item number "S" which corresponds to the starting month for the holding period "P" (measured in months) which is to be analyzed. It is evident that S=E-P--i.e., the starting month must be P months earlier than the ending month. In FIG. 6, which analyzes a one-month holding period, the starting month is one month earlier than the ending month; in FIG. 7, it is 12 months earlier; and in FIG. 8, it is 36 months earlier.

At step 128, CPU 20 computes the change percentage for the target rate values 310, and the fund values 210, for each holding period, starting with the holding period which ends with the ending month for observation one (July '86) and terminating with the ending month for observation 84 (June '93). For example:

(1) In FIG. 6, the first one-month change percentage 170 for the target rate values 310 (col. 5) is determined as follows (wherein the notation 37(4), e.g., means the entry in col. 4 for item 37): ##EQU1##

(2) Similarly, the first one-month change percentage 122 for the fund values 210 (col. 7) is determined as: ##EQU2##

(3) In FIG. 7, the first 12-month change percentage 170 for the target rate values 310 (col. 5) is determined as: ##EQU3##

(4) And the first 12-month change percentage 122 for the fund values 210 (col. 7) is determined as: ##EQU4##

(5) In FIG. 8, the first 36-month change percentage 170 for the target rate values 310 (col. 5) is determined as: ##EQU5##

(6) And the first 36-month change percentage 170 for the fund values (col. 8) is determined as: ##EQU6##

At test 174, CPU 20 then determines whether or not the holding period 410 is greater than one year. If the holding period 410 is greater than one year, change percentage 170 (denoted by C.sub.E below, for convenience) is routed by CPU 20 to annualization step 176, where it is annualized according to the following formula:

Annualized change % 120.sub.E =(C.sub.E+ 1.00000).sup.12/p -1.00000,

where P represents the holding period 410 expressed in months. CPU 20 then tests at step 184 to see whether the value data being processed is fund value data 210 or target rate value data 310. If the data is fund value data, then CPU 20 resumes processing at step 178, as described below. If the data is target value data 310, annualized change percentage 120.sub.E is diverted to subtraction 130, skipping step 178. (Annualized change percentage 120 appears in the sixth column of FIG. 8.)

If, on the other hand, the holding period 410 is determined at test 174 to be less than or equal to one year and test 185 determines that the data being processed is target value data 310, CPU 20 routes change percentage 170 directly to subtraction step 130, skipping step 179. (If the holding period is less than one year, the annualization step is performed later; see below.) And if the holding period is exactly one year, the change percentage 170 is already annualized. Change percentage 170 is now called annualized change percentage 120, the modification in reference numeral made for consistency with any other annualized change percentage 120.

At test 192, CPU 20 checks whether E=84, which condition if true indicates that all 84 observations have been made. If E is determined by test 192 to equal 84, CPU 20 begins processing of the fund

data 210 at step 190.

If E is determined to be less than 84, CPU 20 repeats the same processing sequence at steps 191, 118, 128, 174, 176, 184, 185, 192 for successive pairs of data points (each pair spaced one holding period apart) of target values 330, as index variable (observation) E is incremented at step 191 over the integers 1 to 84. Specifically, at step 128, value 330.sub.S is subtracted from value 330.sub.B and the result is divided by value 330.sub.S to yield target data change percentage 170.sub.E. CPU 20 at step 176 then annualizes the result as described above if at test 174 the holding period 410 is determined to be greater than one year. Steps 191, 118, 128, 174, 176, 184, 185, 192 repeat this process until test 192 determines that the last scheduled ending month 220.sub.E has been reached i.e. E=84. When test 192 so determines, CPU 20 resumes processing at step 190. (In an alternate embodiment, index variable E may be incremented more rapidly at step 191, so that the data are sampled more readily in time. For example, E may be incremented at step 190 by odd numbers 1, 3, 5, etc., to skip the data for all even numbered months.)

After index variable E has been reset to zero, at step 190 and incremented at step 191, the fund value data 210 and holding period 410 stored at register 116 are passed to month selection step 118 and then to subtraction and division step 128 in the same manner as target value data 330 were, thereby producing for each pair of fund values a fund change percentage 122. (Representative fund change percentages 122 are shown in col. 7 of FIGS. 6 and 7 and in col. 8 of FIG. 8.)

CPU 20 then proceeds to test 174 and determines whether the holding period 410 is greater than one year. If the holding period 410 is more than one year, fund change percentage 122 is annualized at step 176 in the same manner as target value data 310 was annualized at step 176, so that an annualized 30 change percentage 123 is formed. After confirming at test 184 that the data being processed is fund value data 210 and not target value data 310, CPU 20 routes change percentage 123 to step 178 where sales load 420 (denoted "L" for notational convenience) if any, is taken into account. If at step 174, CPU 20 determines that the holding period 410 is one year or less (which applies to FIGS. 6 and 7) test 185 routes each fund change percentage 122 (denoted "C") to step 179, where sales load is accounted for by calculating a load-adjusted fund change percentage 124 (denoted "LAC"), by the following formula: LAC=(1.0000+C).times.(1-L)-1.0000. In FIG. 6, for July 1986, e.g., LAC 124 (col. 9) is determined as (1.0000-.0667).times.(1.0000 -.03)-1.0000 which becomes (0.9333).times.(0.97)-1.0000=0.9053-1.0000=-9.47%.

If at step 174, CPU 20 determines that the holding period 410 is more than one year, which applies to FIG. 8, then LAC 124 is calculated at step 178 by the following formula:

LAC=›(1.0000+C).times.(1-L).sup.12/p !-1.0000 (where C denotes the annualized change percentage 123 without recognizing sales load). Here is an example of the first such entry in col. 11 of FIG. 8, for July 1986:

LAC=›(1.0000+0.2077).times.(0.97).sup.1/3 !-1.0000, which becomes ›(1.2077).times.(0.9899)!-1.0000=1.1955-1.0000=19.55%.

Load-adjusted change percentages 124 are then passed to both subtraction step 130 and averaging step 134. CPU 20 tests at step 193 whether all fund value data 230 have been processed, i.e. whether E=84. If E is found to be less than 84, then CPU 20 resumes processing at step 191; otherwise CPU 20 resumes processing at step 130. At step 130, CPU 20 subtracts change percentage 123.sub.E from change percentage 120.sub.E, for all values of E from 1 to 84, with the results, if positive (as determined at step 132), designated 121.sub.E, for shortfall not recognizing any sales load. These results appear in col. 8 of FIGS. 6 and 7 and col. 10 of FIG. 8). For a fund with a sales load, as is the case for FIGS. 6-8, CPU 20 at step 130 also subtracts LAC 124.sup.E from change percentage 120.sub.E, for all values of E from 1 to 84, with the results, if positive (as determined at step 132), designated 125.sub.E for load adjusted shortfall. (These results appear in col. 10 of FIG. 6 and 7 and col. 12 of FIG. 8.) The results are routed to step 134 where CPU 20 computes four totals: (1) the sum of fund change percentages 123; (2) the sum of load adjusted fund change percentages 124; (3) the sum of shortfalls 121; and (4) the sum of load-adjusted shortfalls 125. Each of the 4 sums is divided

by the number of observations, in this case 84, to provide the average rates of return, with (141) and without (142) load adjustment, and of shortfall, also with (143) and without (144) road adjustment. In the case where the holding period 410 is at least one year, the average rates of return 141, 142, and of shortfall 143, 144, are annual rates. (These results appear on the final page of FIGS. 7-8, at the bottom of each corresponding column referred to above, below observation 84.) Where the holding period is less than one year, annualization has not been performed yet; see below.

Processing then continues at step 140. At subtraction step 140, CPU 20 retrieves loss-to-gain aversion weight 450 from register 116, which becomes the multiple to apply to the average rate of shortfall 144, with the product to be subtracted from the average rate of return 142, to produce the average rate of risk-adjusted return 146. The same is done for the corresponding load-adjusted quantities, i.e., CPU 20 forms a product of weight 450 and average rate of shortfall 143, and subtracts the product from average rate of return 141, to yield average rate of load-adjusted, risk-adjusted return 145.

The weighting at step 140 is performed to reflect the indication provided by behavioral economics that individuals are risk-averse and generally view a potential one dollar loss as completely offsetting a potential two dollar gain of the same likelihood. As observations which indicate a shortfall are already included in the average rate of return, subtracting the average rate of shortfall from the average rate of return (weight 450 equals 1.0) results in counting the shortfall twice, thereby comporting with the behavioral economics view of individual sensitivity to potential gain and loss. The more risk-averse the investor, the higher weight 450 should be set, and vice versa.

Finally, CPU 20 tests at step 182 whether the holding period is less than one year. If not, processing is diverted around step 186, since annualization has already been performed. If so, at step 186, the following quantities are annualized: average rates of risk-adjusted return 145, 146, using the following formula:

Annualized rate=(1.0000+A).sup.12/p -1.0000, where "p" is the holding period, expressed in months, and A is the corresponding average rate as measured for the duration of p months. (See bottom of FIG. 6D)

In the case where mutual fund data 210 include data from more than one mutual fund, the risk-adjusted performance for the additional mutual funds is calculated in precisely the same way by CPU 20 at steps 190, 191, 118, 128, 174, 176, 184, 178, 185, 179, 192, 130, 132, 180, 134, 182, 186, and 140. That is, mutual fund values 240 for the second mutual fund replace values 230 in each of the steps listed above to yield a risk-adjusted performance 144 for the second mutual fund. Indeed it will be clear to those skilled in the art that the same can be accomplished for any number of mutual funds, provided that the value data for each fund is included in fund data 210 and the sales load information for each fund is provided. It will also be clear that the system as described above, in which each mutual fund is processed sequentially, is merely illustrative, and that the processing can be done in other ways. For example, the processing of all of the mutual funds could be accomplished in parallel, with any number of tests as well as any number of selection, subtraction, and averaging steps, and the like connected in parallel.

The methods and apparatus described in detail above are completely appropriate only for an investment that is not subject to tax, as one held by a qualified pension plan or an otherwise tax-exempt investor. For taxable investors, the effect of federal income taxes paid by the investor may be assessed by the invention by replacing the fund data 230, 240, 250 and the target rate data 330 inputted into the system 100 by tax adjusted values for each ("tax-adjusted data"). As tax is payable by a taxable investor as long as he retains an investment only on taxable distributions made available by the investment, it is necessary to obtain a history of such distributions separated into: (1) tax-exempt interest, if any, on which no federal tax is payable; (2) taxable interest, dividends, and short-term capital gains, all of which in the United States are taxed as ordinary income, subject to a current maximum federal tax rate of 39.6%; and (3) long-term capital gains, subject to a current federal tax rate in the United States of 28%. For the target rate investment in 3-month T-Bills, the maximum tax rate in the United States is currently 39.6%. (It would not be practical, as a general matter, to also recognize individual state and local income taxes, although it would theoretically be possible, and it

would be feasible to do so on customized analyses of assets of sufficient magnitude to warrant the additional expense.)

The methods and apparatus for evaluating portfolios based on tax-adjusted investment risk and risk-adjusted return, using such tax-adjusted data, would be exactly the same as has already been described for the procedures not recognizing taxes payable by the investor, with the following modifications required to recognize the impact of federal income taxation: (1) any taxable distributions would be reduced by federal taxes, with only the net amount applied for reinvestment; (2) the adjusted tax basis of the investment would include all amounts applied for reinvestment, in addition to the original investment; and (3) the net amount available upon termination of the investment, by sale or by inheritance, would be the gross proceeds reduced by any applicable federal taxes. (Of course, for consistency, the results for the target rate should also be tax-adjusted unless the target rate is based on a tax-exempt investment.) The format for expressing the results of the evaluation would be exactly the same as has already been described for evaluation not recognizing taxation, namely: (a) average annual rate of tax-adjusted return, minus (b) average annual rate of tax-adjusted shortfall, yielding (c) average annual rate of risk-adjusted, tax-adjusted return. All such results should also be load-adjusted, where appropriate.

Such a tax-adjusted procedure would be required, of course, to properly evaluate portfolios which hold municipal bond funds as well as other mutual funds.

At this point, it would be useful to point out the probabilistic nature of the approach followed by the invention in measuring risk or shortfall. FIG. 7, which analyzes a one-year holding period, shows the results of 84 observations, over a span of 7 years, for 12-month periods ending with months from July 1986 through June 1993. From col. 8, which does not recognize sales loads, it is evident that shortfalls occurred in 18, or 21.43%, of the observations. Also, it is evident from col. 8 that the sum of the 18 shortfalls is 243.33%, so the average shortfall, when it occurred, was 1/18 of 243.33% or 13.52%. Accordingly, the likelihood of shortfall, 0.2143, multiplied by the average shortfall, when it occurred, of 13.52% is 2.90% which is the expectation of shortfall, or in less formal language, the average annual rate of shortfall, for the 84 observations. Recognizing sales loads, in col. 10, indicates 19 shortfalls (22.62%) and a sum of the shortfalls of 295.45%, indicating an average shortfall, when it occurs, of 295.45/19 or 15.55%. Accordingly, the expectation of shortfall increases to (0.2262).times.(0.1555) or 3.52%, the average annual rate of shortfall.

FIG. 9 summarizes the analysis of three mutual funds--one no-load, and two with sales loads. Among the insights which the Summary offers are the following:

1. Sales loads reduce returns and may thereby increase risks, especially for short and very short holding periods.

2. A short-term investment in a relatively volatile fund, especially if it has a significant front-end load, is extremely risky, even for funds such as Fidelity Magellan and AIM Constellation, which show very fine average annual rates of load-adjusted, risk-adjusted return for holding periods of one year and 3 years, but, for a holding period of one month, show average annual rates of load-adjusted, risk-adjusted return ranging from -40.91% to -70.37%. (This insight is not even hinted at by either the Modern Portfolio Theory method or the Morningstar method of measuring investment risk.)

3. Even sound analyses of very short holding periods, like one month, do not reveal any useful information applicable to longer holding periods, like 3 years or even one year. In general, volatility and risk do not appear to be highly correlated, for sufficiently long holding periods.

4. For holding periods of 3 years, and even one year, the effect of sales loads on returns, including risk-adjusted returns, may be overwhelmed by net investment performance.

5. The average annual rates of return, shortfall and risk-adjusted return, for holding periods of one year or more, as indicated by the system for a sufficiently large number of observations, seem reasonably stable (although larger studies with longer histories and more funds would be desirable).

6. Where complete information is provided about the return and risk elements of risk-adjusted returns, as in the lower section of FIG. 9, it is very simple for an individual investor to apply a personal loss-to-gain aversion weight. For example, consider the load-adjusted return and risk elements for 84 observations of a one year holding period for the AIM Constellation Fund. Shown in FIG. 9 are an average annual return of 15.43%, reduced by an average annual rate of shortfall of 4.95%, resulting in an average annual load-adjusted, risk-adjusted return of 10.48%. This result, as has been explained above, reflects a loss-to-gain aversion weight of 1.0. A less risk-averse investor, who considers his personal loss-to-gain aversion weight to be only 0.5, should reduce the average annual rate of return (15.43%), by only half of the average annual rate of shortfall (4.95%), or by 2.475%, resulting in a personal annual load-adjusted, risk-adjusted rate of return of 12.955%.

It is a well known and well accepted truism that the past is not necessarily a predictor of the future, particularly for, but not limited to, financial matters. Some observers believe that the past is a better predictor of longer term trends than of shorter term. In any situation there is an excellent case to be made that reliance should not be based solely on the past, but that the implications of any retrospective analysis should be subjected to the influence of prospective views of the future. But no one would suggest that no consideration whatever should be given to the implications of past records. And therefore, for a balanced view, as the prospects of the future are always uncertain, it is useful to obtain the most reliable and informative analysis of the past.

The pioneering work of Harry Markowitz, in 1952, and William F. Sharpe, in 1963, in the development of what has become known as Modern Portfolio Theory (MPT) can properly be characterized as developing the quantitative analysis of investment volatility, or the variability of short-term results. Their work led to growing awareness of the nature of investment volatility and to interest in controlling such volatility, and especially to controlling variance from market results. Accordingly, the fathers of MPT are credited therefore with stimulating the development of index funds, which are now a major factor in individual and institutional investment in stocks and bonds, in the United States and abroad.

The versatility of the invention, in applying a probabilistic approach to measuring investment risk, as distinguished from volatility, to a wide variety of investments of different classes, on a uniform basis, is believed to offer a similar promise: to lead to the development of procedures to manage the risk of portfolios, by controlling the degree of longer-term shortfall, by either static or dynamic systems.

The invention includes, and can be applied to, the selection and optimization of a portfolio of mutual funds, as follows:

1. The client establishes the following objectives:

A. The library of mutual funds from which the portfolio is to be selected consists solely of 12 index funds offered by The Vanguard Group, Inc.: (1) 6 U.S. Stock funds: Total Stock Market; S&P 500; Extended Market; Small Cap; Growth; Value; (2) 3 international stock funds: Europe; Pacific; Emerging Market; and (3) 3 U.S. Bond Funds: Total Bond Market; Intermediate Term Bonds; and Short Term Bonds. By using only Index Funds, the client is assured that the performance of each fund will exhibit absolute fidelity to its investment policy.

B. The portfolio to be selected is to consist solely of 6 Index Funds in the weights resulting from the proportions fixed at the outset (i.e., on a static basis).

C. The portfolio should be optimized to have provided the highest average risk-adjusted rate of return consistent with the risk limitations that (1) the maximum individual event of shortfall did not exceed 15% and (2) the likelihood of a shortfall of 10% did not exceed 5%, for a one-year holding period, as indicated by observations ending in the 84 months from July 1986 through June 1993, and with the target rate of return set at T-Bills, and the loss-to-gain aversion weight set at 1.0. (Aside from the additional risk limitations, these conditions are the same as those which apply to FIG. 7.)

2. For simplicity, this embodiment assumes that the client is either a tax-exempt entity, such as a pension fund, or is otherwise not interested in an analysis which recognizes taxes on the investor. Furthermore, for this purpose, let us assume that these 12 Vanguard funds do not have any sales loads or transaction fees.

3. The first step would be to prepare for each of the 12 funds a table like FIG. 7, but with an additional column 8A. Col. 8A would be similar to Col. 8 in FIG. 7 except that it would be equal to Col. 5 minus Col. 7 without the restriction that applies to Col. 8, namely that the entries must be either positive or zero. In other words, the entries in Col. 8A would be the unrestricted variance from the target rate and therefore could be positive, zero, or negative. Col. 8 would be used, as in FIG. 7, to determine the average annual shortfall of a fund; Col. 8A would be used to determine the likelihood that the experience of the variance of two funds might be sufficiently inversely correlated that their combination could result in lower average annual shortfall than one or both funds experienced individually, i.e. that they would hedge one another to some extent.

4. The correlation of the variance experience of two funds would be determined using the following formula: ##EQU7## where X.sub.i and Y.sub.i are the corresponding variance entries in Col. 8A for two funds at observations for the same ending month, and X and y are the averages, respectively of all variance entries in Col. (8A) for each of the two funds. In general, the lower the correlation of the variance of two funds, especially if it is negative, the greater is the likelihood that they would hedge one another to some extent.

5. The second step would be to determine which of the stock funds should be considered the core stock fund. In view of the high percentage of the total capitalization of U.S. stocks which they represent, both the Total Stock Market fund and the S & P 500 fund are presumptively appropriate candidates to be the core stock fund for a U.S. investor. An examination of the average annual rate of risk-adjusted return of the two funds should indicate which one should be chosen (i.e. the one with the higher average risk-adjusted rate of return).

6. The third step would be to compute the correlation coefficient of the variance of the selected core fund with each of the remaining 4 United States stock funds.

7. By iteration, starting with the fund which has the lowest correlation with the core stock fund, there would be run the equivalent of 4 versions of the expanded FIG. 7 for an initial investment consisting of 50% of the core fund and 50% of each of the other 4 United States stock funds. Examination of the results would indicate for each combination, (a) the degree to which it complies with the risk limitations set by the client, (b) the average annual risk-adjusted return, (c) the degree of similarity to the results for other combinations. For example, the results for the combination with the Extended Market fund would be expected to be somewhat similar to the results with the Small Cap fund. In that case the choice indicated would be that combination with the better average risk-adjusted return, provided the risk limitations are met. In this way, funds would be eliminated, until the best two-fund U.S. stock combination is determined, with the two funds equally weighted at the outset.

8. Having found the best two-fund United States stock combination, the next step would be to determine, in a similar manner, which of the three international stock funds could produce the optimum three-fund stock combination with the two United States stock funds previously selected, with the three funds equally weighted at the outset.

9. After determining the best three-fund combination of stock funds, the next step would be to determine in a similar manner, which United States Bond fund could be added to determine the optimum four-fund combination with the three stock funds previously selected, with the four funds equally weighted at the outset.

10. In a similar manner, there would be determined a fourth stock fund and then a second bond fund, making a total of six funds.

11. With the six funds selected on an equally weighted basis, the next step would be, by iteration, to

ascertain the optimum variable weighting of the six funds that produced the best average annual risk-adjusted return consistent with the risk limitations. This optimization might be performed by using standard local optimization techniques, such as gradient ascent or descent, to locate local maxima or minima.

12. After completion of the preceding analysis, it would be desirable to complement the results derived from the historical experience with any prospective views of the future that may seem appropriate, and run the experience of any revision of the selection and optimization of the portfolio, to observe the implications of the changes.

The foregoing description provides only a small number of examples of the use of the present invention to measure investment risk, or to select and optimize portfolios of mutual funds. Furthermore, as an alternative to implementing the invention by hardware that includes a general purpose computer, the invention may be implemented purely in hardware. For example, to implement the portfolio selector described above, CPU 20 shown in FIG. 1 could be replaced by a combination of hardware components as depicted in FIG. 10.

In FIG. 10, which shows a second preferred embodiment of apparatus according to the invention, the hardware combination 1000 which replaces CPU 20 includes a communication port 1010 for accessing fund value data 210 and target value data 310 from bus 50. Port 1010 feeds fund value data 210 corresponding to the first group into a multiplexer 1014, fund value data 210 corresponding to all remaining groups into another multiplexer 1016, value data 210 and 310 into processor modules 1018 and 1020, and investor preference data 740 into a screening module 1032.

Counter 1022, which counts from 1 to the number of assets in the first group, engages the select lines of multiplexer 1014 so that the multiplexer's output is fund data 210 for the selected fund. Processor module 1018 takes the output of multiplexer 1014, along with the target value data it received from port 1010, and computes an average risk-adjusted return for the selected fund. Counter 1022 continues counting until all funds in the first group have been processed by processor module 1018, at which point the risk-adjusted returns are outputted by processor module 1018 into a comparator 1024. Comparator 1024 chooses the fund in the first group having the highest risk-adjusted return, designates the fund as the tentative portfolio, and outputs the tentative portfolio to combination module 1028.

Next, in response to a second counter 1036, which counts from 2 to the number of groups in the library, multiplexer 1016 feeds the fund value data 210 for the selected group into a correlator 1030 and into combination module 1028. Fund data corresponding to the tentative portfolio is likewise routed to correlator 1030 from combination module 1028. Correlator 1030 computes a correlation of the variances of each fund and the variance of the tentative portfolio. Module 1028 combines the fund data 210 for each fund in the selected group with fund data corresponding to the tentative portfolio, with the combinations of fund data fed to a second processor module 1020, which computes the average risk-adjusted return for each combination.

The output from correlator 1030 is screened by module 1032, which determines which combinations have correlations that satisfy criteria derived from the investor preference data 740 received from bus 50 via communication port 1010. This determination, as well as the output computed by module 1020, is received by comparator 1034, which chooses the combination having the highest average risk-adjusted return from among those passing the screening performed by module 1032, designates the combination as the new tentative portfolio, and feeds the value data for the tentative portfolio back to combination module 1028. As counter 1036 increments, the tentative portfolio is augmented by additional funds in the manner described above, until the last group has been processed. The tentative portfolio is then passed to differentiator module 1038, which optimizes the portfolio by performing gradient ascent to locate a local maximum in average risk-adjusted return. The resulting, optimized portfolio is outputted via an output port 1040 and a communication port 1042.

In an alternative embodiment, some or all of multiplexers 1014 and 1016, processor modules 1018 and 1020, counters 1022 and 1036, comparators 1024 and 1034, correlator 1030, combination

module 1028, screening module 1032, and differentiator 1038 may collectively be implemented by appropriately hardwiring one or more gate arrays.

In yet another embodiment, this collection of hardware could be implemented by a programmable logic device ("PLD"), such as the FLEX 8000.TM. PLD manufactured by Altera Corporation, of San Jose, Calif., coupled to an erasable programmable read-only memory ("EPROM"). One advantage to using a PLD-based hardware system would be the ability to dynamically reconfigure the hardware components.

FIG. 11 shows a cross section of a magnetic data storage medium 1100 which can be encoded with a machine-executable program that can be carried out by a system such as system 10 of FIG. 1. Medium 1100 can be a conventional floppy diskette or hard disk, having a suitable substrate 1101, which may be conventional, and a suitable coating 1102, which may be conventional, on one or both sides, containing magnetic domains (not visible) whose polarity or orientation can be altered magnetically. Medium 1100 may also have an opening (not shown) for receiving the spindle of a disk drive or other data storage device 24.

The magnetic domains of coating 1102 of medium 1100 are polarized or oriented so as to encode, in a manner which may be conventional, a machine-executable program such as that described above in connection with FIG. 5, for execution by a system such as system 10 of FIG. 1.

FIG. 12 shows a cross section of an optically-readable data storage medium 1200 which also can be encoded with such a machine-executable program, which can be carried out by a system such as system 10 of FIG. 1. Medium 1200 can be a conventional compact disk read only memory ("CD-ROM") or a rewritable medium such as a magneto-optical disk which is optically readable and magneto-optically writable. Medium 1200 preferably has a suitable substrate 1201, which may be conventional, and a suitable coating 1202, which may be conventional, usually on one side of substrate 1201.

In the case of a CD-ROM, as is well known, coating 1202 is reflective and is impressed with a plurality of pits 1203 to encode the machine-executable program. The arrangement of pits is read by reflecting laser light off the surface of coating 1202. A protective coating 1204, which preferably is substantially transparent, is provided on top of coating 1202.

In the case of a magneto-optical disk, as is well known, coating 1202 has no pits 1203, but has a plurality of magnetic domains whose polarity or orientation can be changed magnetically when heated above a certain temperature, as by a laser (not shown). The orientation of the domains can be read by measuring the polarization of laser light reflected from coating 1202. The arrangement of the domains encodes the program as described above.

Those skilled in the art will appreciate that the present invention can be practiced by other than the described embodiments, which are presented for the purposes of illustration and not of limitation, and the present invention is limited only by the claims which follow.

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(1 of 1)

United States Patent	**5,126,936**
Champion, et. al.	**Jun. 30, 1992**

Goal-directed financial asset management system

Abstract

A data processing apparatus and method controls and implements a goal-directed financial assets management system. The operative system receives investor deposits at selected levels of correspondence to established capital markets. A proportionality factor, or "market multiple" MM, is established as a measure of correspondence between the account and each market or asset of interest. The operative system periodically enters new account data and adjusts the individual accounts in response thereto. The system determines a net position change which is translated into aggregate purchase/sale orders of various market index ***futures*** contracts or other capital instruments. The system automatically adjusts the risk exposure in any asset category to prevent its reaching an excessive level. As a result, an account can never lose more than the amount deposited. The data processing system provides efficient operation and low transaction fees to the participating investors.

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Field of Search:	**364/408, 401**

References Cited | [Referenced By]

U.S. Patent Documents

3,697,693	Oct., 1972	Deschenes et al.	**364**/408 **X**
4,597,046	Jun., 1986	Musmanno et al.	**364/408**
4,903,201	Feb., 1990	Wagner	**364/408**
4,953,085	Aug., 1990	Atkins	**364/408**

Other References

Eckardt, Walter L., Jr., and Bridge Data Company, The Index Market ***Fund*** (1984).

Mayers, Robert C., "One Day Market Forecaster", Computer Shopper, Nov. 1987, pp. 236, 253-256.

Crawford, Diane, "Software at His Beck and Call, Manager's Bonds are Beautiful", Wall Street Computer Review, Jun. 1985.

Roberts, Steve, "Makes the World Go Round", Wall Street Computer Review, Mar. 1989.

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18 Claims, 16 Drawing Figures

United States Patent **5,237,500**
Perg, et. al. **Aug. 17, 1993**

System and process for converting constant dollar financial instruments

Abstract

The present invention discloses a method and apparatus for converting constant-dollar financial instruments into equivalent nominal-dollar instruments. After the optimal form or forms of constant-dollar financial instruments have been determined for the purposes of financing a specific enterprise or activity, the data describing the constant-dollar financial instrument or instruments are entered into the system together with the specified inflation measure and the desired frequency of adjustments to the nominal-dollar interest rate and to the nominal-dollar payments. The data processing system puts the specified constant-dollar instrument or instruments into a standardized format and, given the desired frequency of inflation adjustments to be made to the nominal-dollar interest rate and to the nominal-dollar payments, the system specifies the equivalent nominal-dollar instrument or instruments in a standardized format. Every payment and/or compounding period the data processing system calculates the nominal-dollar interest rate(s), nominal-dollar payment(s), nominal-dollar call price(s), and remaining nominal-dollar principal balance for the equivalent nominal-dollar instrument or instruments.

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Field of Search:	**364/408**

References Cited | [Referenced By]

U.S. Patent Documents

4,642,768	Feb., 1987	Roberts	**364/408**
4,648,038	Mar., 1987	Roberts et al.	**364/408**
4,722,055	Jan., 1988	Roberts et al.	**364/408**
4,739,478	Apr., 1988	Roberts et al.	**364/408**
4,742,457	May, 1988	Leon et al.	**364/408**
4,752,877	Jun., 1988	Roberts et al.	**364/408**

Other References

"Purchasing-Power Mortgages-New Loan Plan Offers Hope In An Inflation" McDonough, William R., Business Review, Sep. 1975, pp. 1-7.

"The Taxation and Marketing of Price-Level Adjusted Mortgages", Knoll, Michael S., The Journal Of Real Estate Taxation, pp. 291-309.

"Why Are So Few Financial Assets Indexed to Inflation", Weiner, Stuart E. Economic Review, May, 1983 pp. 3-18.

"Indexed Mortgage Pass-Throughs" Sharplin, Arthur D., Pension World, Nov., 1982, pp. 30-33.

"Pivotal Issues", Sharplin, Arthur D., Indexed Lending's, Aug. 1983 pp. 57-63.

"Interest Rates Aren't Really High", Ranson, David, Fortune Jul., 1985

"Affordability and Inflation Protection", McCullock, J. Huston, Mortgage Banking, pp. 8-13.

"Real Estate Investment Analysis System", IBM Technical Disclosure Bulletin vol. 13, No. 11, Apr. 1971.

"Software Packages Assist Diverse Needs of Bond Portfolio Managers" Technical Data Corp. Jun., 1985 Wall Street Computer Review.

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Attorney, Agent or Firm: Banner, Birch, McKie & Beckett

52 Claims, 24 Drawing Figures

United States Patent **5,132,899**
Fox **Jul. 21, 1992**

Stock and cash portfolio development system

Abstract

The present invention combines data gathering and processing methodology with computer apparatus to produce a system whereby a list of stocks and a cash position is generated and purchased for investment and operating accounts. Specifically, the system integrates three areas of data: investment performance for investment managers (the investment manager database); federal Securities Exchange Commission (SEC) reports filed quarterly by investment managers (the government report database); and financial characteristics for a large number of stocks (the stock database). Various screens and criteria are applied to the three data areas. The investment managers in the investment manager database are screened to find investment managers with top performances who meet a series of other criteria. The government reports are screened based upon the largest stock holdings for the investment managers chosen in the first step. The stock database financial characteristics are applied against the stocks from the government reports.

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Filed: **Oct. 16, 1989**
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Current U.S. Cl.: **705/36**
Field of Search: 364/408, 400, 401, 900

References Cited | [Referenced By]

U.S. Patent Documents

4,334,270	Jun., 1982	Towers	364/408
4,597,046	Jun., 1986	Musmanno et al.	364/408
4,823,265	Apr., 1989	Nelson	364/408

Other References

"Software Packages Assist Diverse Needs of Bond Portfolio Managers" Wall Street Computer Review Jun. 85 pp. 61-65.

"Insider Trading Databases" Satish Tyagi Wall Street Computer Review Jun. 85 pp. 53-56.

"Portfolio Management Systems" Steve Roberst Wallstreet Computer Review Mar. 89.

"The electric Wizard at Wall Street" Dec. 1984 Marketing Communications pp. 32-34.

United States Patent **4,953,085**
Atkins **Aug. 28, 1990**

System for the operation of a financial account

Abstract

A personal financial management program is disclosed incorporating means of implementing, coordinating, supervising, analyzing and reporting upon investments in an array of asset accounts and credit facilities within a client account. Through a mathematical programming function the client specifies his financial objectives, his risk preference, forecast of economic and financial variables, and budgetary constraints. The mathematical programming function suggests to the client a portfolio of investment and credit facilities to best realize his financial objectives over a defined time horizon. In the preferred embodiment the central structural element of the financial account is a mortgage secured by the client's home and one or more asset accounts. Client funds that would normally be used to amortize the mortgage may be alternatively used to increase the value of a designated asset account. The client account is imbalanced if the client's borrowing power is less than the minimum borrowing power specified by the financial institution. If the account is imbalanced, the client may reallocate the distribution of assets and liabilities within the client account and/or modify a set of constraints on the client account. If the client account is still not balanced after modification of the account, the system initiates a liquidation procedure.

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Filed: **Apr. 15, 1987**

Intl. Cl. :	**G06F 15/21**
Current U.S. Cl.:	**705/36; 705/38**
Field of Search:	**364/408, 402**

References Cited | [Referenced By]

U.S. Patent Documents

3,697,693	Oct., 1972	Deschenes	**364/408**
4,376,978	Mar., 1983	Musmanno	**364/408**
4,597,046	Jun., 1986	Musmanno et al.	**364/408**
4,722,055	Jan., 1988	Roberts	**364/408**

Other References

Clay, John R. Kaun, "Guide to Personal Financial Planning", 1986.

Woodwell, Donald R., "Automating your Financial Portfolio", 1986.

IBM, "Fund Allocation System", 1985.

Execucom Systems Corporation, "Analysis Extension", 1986.

Personal Computing, "Dialing for Profits", May 1986.

Business Week "A Financial Planner with Nerves of Silicon", p. 108, Oct. 7, 1985.

Institutional Investor, "Hearing about Artifical Intelligence", Jul. 1986, p. 208.

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33 Claims, 16 Drawing Figures

(1 of 1)

United States Patent *5,819,237*
Garman **October 6, 1998**

System and method for determination of incremental value at risk for securities trading

Abstract

A system, method, and product determines the incremental impact of any number of candidate trades on the value at risk (VaR) measure of a trading portfolio within a trading interval, without requiring that the VaR measure be redetermined individually with respect to each candidate trade. The method includes determining the VaR measure for the trading portfolio, and determining a derivative vector quantity for the VaR measure. For each candidate trade, the impact of the candidate trade on the VaR measure is determined as the vector product of the derivative vector and the mapped cashflows of the candidate trade. A negative sign indicates a desirable reduction in the VaR measure. This determination may be made for any number of candidate trades without having to re-determine the VaR measure. The software product employs this method in a financial analysis application in an optimized implementation. The system includes the software product along with databases storing the trading portfolio(s). Additionally, the method and product allow each candidate trade to be normalized with respect to selected criteria, so that a number of individual candidate trades may be ranked with respect to their incremental impact on the VaR measure to determine the candidate trade the best reduces the VaR measure.

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Appl. No.: **600685**
Filed: **February 13, 1996**

U.S. Class:	**705/36**
Intern'l Class:	G06F 017/60
Field of Search:	705/35,36,37,38

References Cited [Referenced By]

U.S. Patent Documents

5446885	Aug., 1995	Moore et al..

Foreign Patent Documents

401203	May., 1990	EP.
WO96/30850	Oct., 1996	WO.

Other References

Beckstrom, Rod and Campbell, Alyce, "An Introduction to VaR", 1995 C.cndot.ATS Software, Inc.
Kalra, Anu, "Measuring Value-at-Risk", Learning Curve, Derivatives Week, Sep. 26, 1994.
Morgan, J.P., "Latest Developments in RiskMetrics", Risk Magazine Annual Congress, Morgan Guaranty Trust Company, Jan. 25, 1996.
Morgan, J.P., "Introduction to RiskMetrics", Morgan Guaranty Trust Company, Market Risk Research, Oct. 6, 1994.
Morgan, J.P., "RiskMetrics.TM.--Technical Document", Third Edition, Morgan Guaranty Trust Company, Global Research, May 26, 1995.

Primary Examiner: Weinhardt; Robert A.
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Attorney, Agent or Firm: Fenwick & West LLP

Claims

1. A computer readable memory storing thereon a computer program for controlling the execution of a processor to determine whether any candidate trade in a selected trading interval reduces a value at risk measure of a trading portfolio having a selected financial instruments, the computer program controlling the processor to:

determine the value at risk measure for the trading portfolio once with respect to the trading interval;

determine a derivative vector of the value at risk measure;

determine a set of cashflows for the candidate trade;

determine an incremental value at risk measure for a candidate trade from the derivative vector of the value at risk measure and the set of cashflows for the candidate trade; and,

determine from the sign of the incremental value at risk measure whether the value at risk measure of the trading portfolio is reduced by the candidate trade.

2. The computer readable memory of claim 1, wherein to determine the derivative vector of the value at risk measure for the trading portfolio, the computer program thereon controls the processor to:

convert the financial instruments in the trading portfolio into a first set of cashflows;

map the first set of cashflows unto a set of selected tenors;

determine a vector product W of transposed mapped cashflows and a scaled variance-covariance matrix of risk variances for the financial instruments in the trading portfolio at the selected tenors;

determine the value at risk measure from the square root of the inner product of W and the variance-covariance matrix; and,

determine the derivative vector from the quotient of W and the value at risk measure.

3. The computer readable memory of claim 2, wherein the computer program thereon controls the processor to:

receive a plurality of candidate trades, each candidate trade including of at least one financial instrument;

for each candidate trade:

convert the candidate trade into a second set of cashflows;

map the second set of cashflows unto the set of selected tenors;

normalize the mapped second set of cashflows for the candidate trade by a normalization factor;

determine an incremental value at risk measure from the derivative vector and the mapped second set of cashflows for the candidate trade; and,

rank each candidate trade by its incremental value at risk measure.

4. The computer readable memory of claim 3, wherein the normalization factor is:

a value at risk normalization factor determined according to the equation: ##EQU6##
where .lambda..sub.i is the value at risk normalization factor, and a.sub.i is the second set of cashflows.

5. The computer readable memory of claim 3, wherein the normalization factor is:

a return normalization factor equal to the value of the anticipated future returns of the candidate trade.

6. The computer readable memory of claim 3, wherein the normalization factor is:

a capital normalization factor equal to an amount of capital required sustain the candidate trade.

7. The computer readable memory of claim 3, wherein the normalization factor is:

a price normalization factor equal to a market price of the candidate trade.

8. The computer readable memory of claim 3, wherein the normalization factor is:

a notional normalization factor equal to a predetermined number of units for the candidate trade selected according to the type of the candidate trade.

9. The computer readable memory of claim 3, wherein the normalization factor is:

a cashflow normalization factor determined according to the equation: ##EQU7##
where .lambda..sub.i is the cashflow normalization factor, and a.sub.i is the second set of cashflows, and {c.sub.j } is an arbitrary set of positive weights.

10. The computer readable memory of claim 3, wherein the normalization factor is:

a cashflow normalization factor determined according to the equation: ##EQU8##
where .lambda..sub.i is the cashflow normalization factor, a.sub.i is the second set of cashflows, and {c.sub.j } is an arbitrary set of positive weights.

11. The computer readable memory of claim 3, wherein the normalization factor is:

a cashflow normalization factor determined according to the equation:

.lambda..sub.i =.vertline..vertline.a.sub.i .vertline..vertline..ident.max.sub.j

{c.sub.j .vertline.a.sub.ij .vertline.}

where .lambda..sub.i is the cashflow normalization factor, a.sub.i is the second set of cashflows, and {c.sub.j } is an arbitrary set of positive weights.

12. The computer readable memory of claim 2 wherein the computer program thereon controls the processor to:

receive at least one candidate trade of at least one financial instrument;

convert the candidate trade into a second set of cashflows;

map the second set of cashflows unto the set of selected tenors; and,

determine an incremental value at risk measure as the product of the derivative vector and the mapped second set of cashflows.

13. The computer readable memory of claim 2, wherein the computer program stored thereon controls the processor to:

determine the vector product W of transposed mapped cashflows p and the selected variance-covariance matrix Q according to the equation:

W=p'Q;

determine the value at risk measure, VaR, according to the equation:

VaR=Wp;

prior to determining the derivative vector, DELVAR, according to the equation:

DELVAR=W/Var.

14.

14. The computer readable memory of claim 1, wherein to determine the incremental value at risk for the selected set of candidate trades, the computer program thereon controls the processor to:

determine an incremental value at risk for each candidate trade in the selected set of candidate trades from the derivative vector and the set of cashflows for the candidate trade; and

accumulate the incremental value at risk for all of the candidate trades.

15. A computer readable memory storing thereon a computer program for controlling the execution of a processor to determine an incremental impact of a candidate trade on a value at risk measure of a trading portfolio P having selected financial instruments, the computer program controlling the processor to:

determine the value at risk measure VaR once for the trading portfolio for a selected trading interval;

determine the derivative vector of the value at risk measure VaR once for the trading portfolio for the selected trading interval;

determine a set of cashflows for the candidate trade;

determine the incremental impact of the candidate trade on the value at risk measure VaR from the product of the derivative vector and the set of cashflows for the candidate trade.

16. The computer readable memory of claim 15, coupled to an article of manufacture including:

a storage device storing at least one trading portfolio comprised of a plurality of financial instruments, and a selected scaled variance-covariance matrix Q of market risk variances for a plurality of financial instruments including the selected financial instruments, the market risk variances determined with respect to a set of selected tenors;

a processor coupled to the storage device, and controlled by the computer program stored in the memory to determine the value at risk measure VaR once for the trading portfolio for the selected trading interval by:

converting the financial instruments in the selected trading portfolio into a first set of cashflows;

mapping the first set of cashflows unto the set of selected tenors;

determining a vector product W of transposed mapped cashflows and the selected variance-covariance matrix; and,

determining the value at risk measure VaR from as square root of the inner product of the mapped cashflows and the vector product W.

17. The computer readable memory of claim 16, wherein the computer program stored thereon controls the processor to determine the derivative vector of the value at risk measure VaR as the quotient of the vector product W divided by the value at risk measure VaR.

18. The computer readable memory of claim 15, the computer program stored thereon controlling the processor to:

receive a selected plurality of candidate trades; for each candidate trade:

determine a set of cashflows for the candidate trade;

normalize the set of cashflows by a normalization factor;

determine for the candidate trade an incremental impact value on the value at risk measure VaR from the product of the derivative vector and the normalized set of cashflows of the candidate trade; and,

rank the selected plurality of candidate trades by the incremental impact values.

19. A computer implemented method of determining whether any candidate trade in a selected trading interval reduces a value at risk measure of a trading portfolio, comprising:

determining the value at risk measure for the trading portfolio once with respect to the trading interval;

determining a derivative vector of the value at risk measure;

determining a set of cashflows for the candidate trade;

determining an incremental value at risk for a candidate trade from the derivative vector and the set of cashflows for the candidate trade; and,

determining from the sign of the incremental value at risk whether the candidate trade reduces the value at risk measure of the trading portfolio.

20. The computer implemented method of claim 19, further comprising:

receiving a plurality of candidate trades, each candidate trade including of at least one financial instrument; for each candidate trade:

normalizing the candidate trade by a normalization factor;

determining an incremental value at risk from the derivative vector and the normalized candidate trade; and,

ranking each candidate trade by its incremental value at risk to determine at least one of the candidate trades that most reduce the value at risk measure of the trading portfolio.

21. The computer implemented method of claim 20, further comprising for each candidate trade:

converting the candidate trade into a second set of cashflows;

mapping the second set of cashflows unto the set of selected tenors;

normalizing the mapped second set of cashflows for the candidate trade by the normalization factor; and,

determining an incremental value at risk from the derivative vector and the mapped second set of cashflows for the candidate trade.

22. The computer implemented method of claim 20, wherein the normalization factor is:

a value at risk normalization factor determined according to the equation: ##EQU9##
where .lambda..sub.i is the value at risk normalization factor, and a.sub.i is the second set of cashflows.

23. The computer implemented method of claim 20, wherein the normalization factor is:

a return normalization factor equal to the value of the anticipated future returns of the candidate trade.

24. The computer implemented method of claim 20, wherein the normalization factor is:

a capital normalization factor equal to an amount of capital required sustain the candidate trade.

25. The computer implemented method of claim 20, wherein the normalization factor is:

a price normalization factor equal to a market price of the candidate trade.

26. The computer implemented method of claim 20, wherein the normalization factor is:

a notional normalization factor equal to a predetermined number of units for the candidate trade selected according to the type of the candidate trade.

27. The computer implemented method of claim 20, wherein the normalization factor is:

a cashflow normalization factor determined according to the equation: ##EQU10##
where .lambda..sub.l is the cashflow normalization factor, a.sub.i is the second set of cashflows, and {c.sub.j } is an arbitrary set of positive weights.

28. The computer implemented method of claim 20, wherein the normalization factor is:

a cashflow normalization factor determined according to the equation:

.lambda..sub.i
=.vertline..vertline.a.sub.i .vertline..vertline..ident..SIGMA.c.sub.j .vertline.a.sub.ij .vertline.

where .lambda..sub.i is the cashflow normalization factor, a.sub.i is the second set of cashflows, and {c.sub.j } is an arbitrary set of positive weights.

29. The computer implemented method of claim 20, wherein the normalization factor is:

a cashflow normalization factor determined according to the equation:

.lambda..sub.i =.vertline..vertline.a.sub.i .vertline..vertline..ident.max.sub.j
{c.sub.j .vertline.a.sub.ij .vertline.}

where .lambda..sub.i is the cashflow normalization factor, a.sub.i is the second set of cashflows, and {c.sub.j } is an arbitrary set of positive weights.

30. The computer implemented method of claim 19, wherein determining the value at risk measure comprises:

receiving a scaled variance-covariance matrix of market risk variances for a plurality of financial instruments including selected financial instruments in the trading portfolio, the market risk variances determined with respect to a set of selected tenors;

converting the financial instruments in the trading portfolio into a first set of cashflows;

mapping the first set of cashflows unto the set of selected tenors;

determining a vector product W of transposed mapped cashflows and the selected variance-covariance matrix; and

determining the value at risk measure from the mapped cashflows and the vector product W.

31. A computer implemented method of determining an incremental value at risk for a selected set of candidate trades in a trading portfolio during a selected trading interval, comprising:

determining a value at risk measure for the trading portfolio once with respect to the trading interval;

determining a derivative vector of the value at risk measure;

determining a set of cashflows for the selected set of candidate trades;

determining the incremental value at risk for the selected set of candidate trades from the derivative vector and the set of cashflows for the selected set of candidate trades.

32. The method of claim 31, wherein determining the incremental value at risk for the selected set of candidate trades further comprises:

determining an incremental value at risk for each candidate trade in the selected set of candidate trades from the derivative vector and the set of cashflows for the candidate trade; and

accumulating the incremental value at risk for all of the candidate trades.

33. A computer readable memory storing thereon a computer program for controlling the execution of a processor to determine an incremental value at risk for a selected set of candidate trades in a trading portfolio during a selected trading interval, the computer program controlling the processor to:

determine a value at risk measure for the trading portfolio once with respect to the trading interval;

determine a derivative vector of the value at risk measure;

determine a set of cashflows for the selected set of candidate trades;

determine the incremental value at risk for the selected set of candidate trades from the derivative vector and the set of cashflows for the selected set of candidate trades.

Description

BACKGROUND

1. Field of Invention

The present invention relates generally to systems and methods for risk analysis in securities portfolios, and more particularly, to systems and methods for rapid or real time determination of Value at Risk metrics.

2. Background

Value-at-Risk (VaR) is a method for assessment of market-based financial risk in the trading of financial instruments. Given a trading portfolio of financial instruments, and description of the market variance characteristics, a VaR analysis statically determines how much of the value of the trading portfolio might be lost over a given period of time with a given level of probability. This determination is expressed as the "VaR" measure.

FIG. 1 is a data flow diagram of the conventional technique for determining the VaR of an existing trading portfolio. In a conventional VaR system, a trading portfolio P of financial instruments is decomposed into a series of component asset flows (known generically as "cashflows," even though the term "cash" sugguests a particular asset). This process is termed "shredding," and produces a set of cashflows that approximates the current value and risk behavior of the portfolio. The cashflows are then mapped onto a set of specified, benchmark cashflows made at specified future time intervals from the present. The future time intervals are known as "tenors" and the combination of an cashflow type (e.g., US dollars, Deutschemark, gold, and the like) and a tenor is termed a "vertex." The mapping is useful in order to provide a representation of the portfolio as a standardized collection of cashflows. The vertices onto which the cashflow set are mapped are those also used in a variance-covariance matrix Q of the market values of the benchmark cashflows. The covariance matrix Q describes the current market characteristics to a reasonable degree of detail. The shredding and mapping creates a set p of mapped cashflows from a portfolio P. These cashflows are then subjected to arithmetic operations with covariance matrix Q to produce the VaR measure.

For example, suppose that the trading portfolio includes financial instruments maturing in arbitrary number of days from the present, such as 22 days. The covariance matrix Q typically includes only vertices for other maturation periods of the given financial instrument, such as at 7, 30, and 60 days from the present. In order to reliably determine VaR in a conventional manner, the financial instrument is then mapped into selected cashflows at the vertices, for example, at either 7 or 30 days, or some distribution there between. There are a number of mapping and shredding functions available to create the set of mapped cashflows p, and the selection of such functions is not relevant to the present invention.

From the mapped cashflows, the VaR of the portfolio is determined by taking the square root of the product of the transpose p' of the set of mapped cashflows, the covariance matrix Q, and the original mapped cashflows p. The resulting VaR value specifies how much money a trader might lose in the current trading portfolio over a given interval of time, with a given probability.

For example, a financial instrument known as a "currency swap" may consist of the promise to pay certain amounts of Deutschemark in return for receiving certain amounts of U.S. dollars, at certain times. Shredding reduces the currency swap into some set of cashflows, being, for example, negative in sign for the Deutschemark flows, positive in sign for the U.S. dollar flows. These shredded cashflows are each scheduled to occur at some assigned point in time in the future, as determined by the swap contract itself. To measure the market risk of the swap, the market risk of a benchmark set of cashflows is determined, for example, for $1 received (or paid) today, in one week, in one month, in 3 months, 6 months, 1 year, and so forth, and similarly for 1 DM received (or paid) at the same tenors. The risks here are determined in part by the variances and covariances of all these quantities at the selected tenors, and in part by the amounts of such benchmark (vertex) cashflows (Risks are measured only at benchmark tenors because measuring variances and covariances for all possible cashflows at all possible arbitrary tenors would be computationally infeasible.) However, the shredded cashflows of the original swap contract do not necessarily lie exactly upon these vertices where the benchmark risks were measured. Therefore, the shredded cashflows are next "mapped" onto the vertices, in amounts that behave equivalently in terms of risk.

In the currency swap example, the set of shredded cashflows is mapped onto "equivalent-sized" cashflows lying at the vertices. As a final step, the risk of all mapped cashflows is calculated together using the known VaR equation, accounting for the risk offsets of low covariance. A more complete discussion of the VaR methodology may be found in RiskMetrics Technical Document (3rd Ed. 1995), the primary source on the method by J. P. Morgan, and in An Introduction to VAR, by C.ATS Software Inc. (Palo Alto, Calif.).

As currently used, the VaR only informs about the current risk characteristics for a given trading portfolio. A natural outgrowth of the calculation of VaR is the question "How can VaR be improved (decreased)?" There are several reasons that make this determination important. Among these, various regulatory and quasi-regulatory bodies (e.g. Bureau of International Settlements) have suggested that VaR should be tied to capital adequacy, i.e. the amount of capital which ought to be required to support certain types and amounts of trading. Thus, financial institutions desire to know the potential effects on VaR from possible trades, preferably in real time, in order to minimize the VaR for their trading portfolios.

Dealing with VaR on a real-time basis is a serious issue for most trading institutions. The typical financial institution may have tens of thousands of financial contracts ("trades"), and each trade gives rise to dozens of shredded cashflows. The number of cashflow vertices may also be as large as 300-1000 (since it represents the product of the number of markets times the number of tenors times the number of asset types). During a trading period, a single trader, perhaps one of hundreds of traders, or even thousands in an institution, may consider dozens of alternative trades during a trading day. It then becomes desirable for each trader to know which of his potential trades will improve (or reduce) the VaR measure for the entire financial institution's trading portfolio. That is, it is desirable to determine the incremental effect of each trade on the VaR measure of a trading portfolio.

In the current art, the only means of evaluating the incremental impact on the VaR of a new, candidate trade is to re-perform the entire VaR analysis on the combination of the existing trading portfolio and the candidate trade. This process is illustrated in FIG. 1, the dashed area, by repeating the VaR calculation for the candidate trade. More specifically, the re-determination of the VaR measure for an individual candidate trade A.sub.(i), is made by merging the candidate A.sub.(i) with the trading portfolio P, shredding and mapping the resulting combined portfolio, and recalculating the VaR.sub.(i) measure. The difference between the original VaR measure and the current VaR.sub.(i) measure is then taken. If the new VaR.sub.(i) measure is less than the original VaR measure, the candidate trade A.sub.(i) will improve the VaR of the trading portfolio. This existing art, however makes it extremely difficult to determine VaR in real time for each potential trade, due to the computationally-intensive nature of such VaR re-calculation.

Moreover, the VaR method is a one-directional calculation: much information is lost in the course of reducing a portfolio of trades into a single VaR number, and the process is not reversible. For

example, many trades might give rise to a given pattern of mapped cashflows; many mapped cashflow patterns might give rise to a given VaR measure; and so on. There is really no direct means of recovering trade-related information from a final VaR measure. As a consequence, conventional systems provide no means of calculating from the VaR measure itself which trades should be done in order to improve the VaR measure, because the process if not reversible: one cannot recover cashflows from VaR numbers, nor trades from cashflows. In sum, the current VaR methodology, being unidirectional in its approach, cannot directly provide information in the opposite direction, that is, determine which trades will reduce the VaR measure.

Finally, VaR is a nonlinear risk measurement which depends not only upon the incremental trade, but also upon how this trade interacts and offsets with the existing portfolio of trades. This interdependence makes it quite difficult to establish the incremental effect of a new proposed trade, per se.

For these reasons, conventional VaR analysis is limited to at best repeatedly recalculating the VaR measure for the entire trading portfolio for every new candidate trade. The process is currently extremely time consuming, given, as noted above, the extremely large number of trades and cashflows that must be analyzed for each and every potential trade. The inability to recalculate VaR measure in real time seriously limits the potential feedback to individual traders and risk managers, who ideally should instead be provided instant response on whether their proposed trades are VaR-improving or not.

Accordingly, it is desirable to provide a system and method for determining incremental VaR measure for each candidate trade, without having to re-examine the institution's entire trading portfolio and recalculate the VaR measure on the combined trading portfolio. In addition, it is desirable to provide a system and method for determining a trade-independent means of comparing various candidate trades quickly and reliably, to identify a trade or trades that best reduce the VaR measure of a trading portfolio.

SUMMARY OF THE INVENTION

The present invention overcomes the limitations of the conventional VaR analysis systems by providing a system, method, and product with which any new proposed trade may be quickly examined for its incremental impact on the VaR measure of the trading portfolio, without further reference to the existing portfolio held by the institution, and without the need to re-determine the combined portfolio's VaR measure for each candidate trade. In addition, the present invention allows a set of candidate trades to be evaluated and ranked to identify the trade(s) that most favorably reduce the VaR measure.

In one aspect the present invention is a financial analysis application, a software product, that operates with conventional computerized financial trading systems. The financial analysis application analyzes a trading portfolio of financial instruments stored in a database or other storage mechanism to produce a VaR measure for the trading portfolio. An improved method is employed that determines not just the VaR measure for the trading portfolio, but further, a vector of intermediate data values here called the "DELVAR." The DELVAR vector is then employed in forming approximations of the rate of change in the VaR measure of a trading portfolio, for any subsequent trade. The financial analysis software need only determine the DELVAR vector once for a given trading portfolio within a selected trading period, such as a single day. With the DELVAR vector, the impact, or incremental VaR of any candidate trade in the trading period may be determined without the computationally expensive process of re-determining the VaR measure for each combination of the existing portfolio and a candidate trade, as in conventional systems.

Accordingly, in this aspect of the invention, a list of candidate trades is provided to the financial analysis application, and each is individually assessed for its impact on the VaR by applying the candidate trade to the DELVAR. The software does this by conventional shredding and mapping each candidate trade to produce a set of mapped cashflows for the trade, typically represented as a vector quantity; the cashflow vector for a candidate trade is then combined with the DELVAR vector to

produce an approximate incremental impact value for the candidate trade.

The DELVAR vector may be computed quite efficiently, using conventional data processing operations. Once the DELVAR vector is determined, an incremental VaR impact value can be computed, which shows whether the VaR for the trading portfolio will change positively or negatively by the addition of the candidate trade to the portfolio of trades. This permits, among its other uses, an incremental analysis of a trader's next proposed trade, without re-examining the institution's entire portfolio.

As a further improvement provided by the present invention, the cashflows of each candidate trade may normalized prior to application to the DELVAR with respect to a given criteria function. Normalization allows any number of candidate trades, even of different types, to be compared according to their incremental VaR impact values. As a result, candidate trades that best reduce the VaR measure of a trading portfolio may be selected and executed, thereby desirably, and expediently reducing the risk of a trading portfolio. Normalization is provided by additional normalization components in the financial analysis application, and by the methods such components employ.

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. 1 is an illustration of the dataflow for a conventional method of determining VaR for a trading portfolio.

FIG. 2 is an illustration one embodiment of a financial analysis system in accordance with present invention.

FIG. 3 is an illustration of one embodiment of the dataflow of the financial analysis system of the present invention.

DETAILED DESCRIPTION OF THE INVENTION

Referring now to FIG. 2, there is shown an illustration of a financial analysis system 100 operating in accordance with one embodiment of the present invention. The system 100 includes a computer 110 coupled to a database 120 via a network 150. The computer 110 is of conventional design, and includes a processor 111, randomly addressable memory (RAM) 112, network interface 113, input and output devices 114, hard disk 115, and a display 116. The computer 110 executes a conventional operating system 117. In a preferred embodiment, the computer is an Intel-compatible personal computer operating Microsoft's Windows.TM. operating system. Other computers and operating systems may also be used.

The computer 110 interfaces with the database 120 on a mass storage device in a conventional manner, to store and retrieve data therefrom. The database 120 stores at least one trading portfolio 130. A trading portfolio 130 is comprised of a set of financial instruments 135 or trades, or other trading portfolios 130. For each financial instrument 135, appropriate data identifying the type, amount, and realization dates, and other attributes of the instrument, along with some means for associating the financial instrument with its trading portfolio 130. In a preferred embodiment, the database 120 is relational. In addition, the database 120 preferably maintains as one of its tables (or other useful data structure) a list of the types of financial instruments 135 used in the portfolios 130, along with a nominal (e.g. $1) amount for the instrument.

In a preferred embodiment, the computer 110 interfaces with a network 150 to communicate with remote other computer systems, and receive from at least one such computer 165 a computer readable dataset comprising a covariance matrix 170 for selected ones of the types of financial instruments 135 stored in the database 120, and a vertex set 175. The covariance matrix 170 and vertex set 175 is then stored in the database 120 or used directly in the RAM 112. The covariance matrix 170 defines the covariance values for the selected financial instruments 135 at predetermined tenors, in the manner described above. The vertex set 175 defines vertices for the tenors in the covariance matrix 170. In one preferred embodiment, the covariance matrix 170 is received and updated to the database 120 on

a daily basis. One source of the covariance matrix 170 and vertex set 175 is J. P. Morgan's RiskMetrics.TM. dataset, available daily on the World Wide Web at:

http: / /www.jpmorgan/RiskMetrics/DOWNLOADING/download-data.html.

Alternatively, the covariance matrix 170 and vertex set 175 may be locally generated.

The database 120 stores user inputs, such as from a trader, of a set 180 of candidate trades 180 of selected financial instruments 135 (the set may include a single candidate trade 180. The manner in which the set of candidate trades 180 is generated is determined by the user of the system 100, and is not restricted by the present invention, which may operate upon any set of candidate trades 180. For each of candidate trades 180 in the set, the system 100 will determine its incremental impact on a VaR of a trading portfolio 130. Optionally, the system 100 may determine from among the set the candidate trades 180 that best improves the VaR of a trading portfolio 130.

The computer 110 stores and executes a financial analysis application 140. The financial analysis application 140 is a software product that operates in accordance with the present invention. The financial analysis application 140 reads and manipulates data regarding a trading portfolio 130 from the database 120 in accordance with the present invention to produce both a DELVAR value for a trading portfolio 130 and an incremental VaR impact value for any candidate trade. The financial analysis application 140 includes a shred/map ("MAP") module 200, a DELVAR module 210, a set of normalization modules 220, and an incremental VaR module 230. While one computer 110 is shown with the financial analysis application 140, additional computers 110 may also be used, each executing directly, or via a client-server relationship, a financial analysis application 140 as described herein.

Referring now to FIG. 3, there is shown a dataflow diagram of the operation of the financial analysis application 140 in accordance with one embodiment of the present invention. First, the VaR value for any number or collection of trading portfolios 130 is determined. A trader, risk manager, or officer selects 300 a trading portfolio 130 for analysis. As a trading portfolio 130 may contain other trading portfolios 130, including the overall position of an institution, the VaR of any aggregation of financial instruments 135 may be determined. Preferably, the VaR for a trading portfolio 130 is done overnight during the non-trading hours, due the time required. It is also preferred that VaR of the trading portfolio 130 for the entire financial institution be determined.

The financial analysis application 140 reads the selected trading portfolio 130 and applies 302 the shred and map functions 200 thereto. These may include any conventional shredding function to produce a set of cashflows for the trading portfolio 135. The financial analysis application 140 reads the covariance matrix 170 and the vertex set 175, and maps each of the cashflows onto a vertex therein according to a predetermined, or user selected mapping function provided by the map module 200. Any variety of shredding or mapping functions may be employed with the system of the present invention. The result of the shred and map module 200 is a set of mapped cashflows 330 for the trading portfolio 130. The mapped cashflows 330 may be characterized as a column vector, in which each vector component is a cashflow. For later convenience in notation, the mapped cashflows 330 are also designated p.

Using the mapped cashflows 330, the VaR measure for the selected trading portfolio 130, and along with the DELVAR vector, are determined by the DELVAR module 210. Unlike conventional VaR systems, the DELVAR module 210 determines the VaR measure in a manner that is optimized for the determination of the DELVAR vector. In one implementation of the DELVAR module 210, operating as illustrated in FIG. 3, an intermediate product W is taken 306 as the vector sum of the transpose p',(304) of the mapped cashflows 330 of the trading portfolio 130. The VaR measure for the selected trading portfolio 130 is then determined 308 as the square root of the inner product of W and p. This VaR determination is only made once for the trading portfolio 130 in a selected time interval, like a trading day or say, 3-hour period. The DELVAR vector is then determined 310 as the quotient of the intermediate quantity W, and the VaR measure, again, only once during the selected trading interval.

The DELVAR module 210 is efficiently implemented by calculating the vector product W of the mapped cashflows 330 and the covariance matrix 170 and storing it as an intermediate step to determining the VaR measure for the trading portfolio 130. The following pseudo-code is an example:

```
SUBROUTINE DELVAR (P, Q, NUM.sub.-- VERTICES)
FOR I = 1 TO NUM.sub.-- VERTICES {
W>I! =0
FOR J = 1 TO NUM.sub.-- VERTICES {
        W>I! = W>I! + P>J! * Q>J,I!
}
}
VAR = SQRT(INNERPROD(W, P, NUM.sub.-- VERTICES))
FOR I = 1 TO NUM.sub.-- VERTICES {
DELVAR>I! = W>I!/VAR
}
}
```

Here, P is the mapped cashflow 330, Q is the scaled covariance matrix 170, NUM.sub.-- VERTICES is provided by the vertex set 175, and INNERPRODO () is a standard calculation of the inner product of two vectors of compatible size. By first determining the row vector W, it is possible to determine DELVAR. It is important to note the DELVAR here was determined without any reference to any candidate trade.

Conventional approaches to VaR, in order to reduce the run time required, do not separately determine W. As a result, conventional VaR calculations requires approximately N.sup.2 steps (where N is shorthand for NUM.sub.-- VERTICES) and no additional memory, while the DELVAR module 220 requires approximately N.sup.2 +N steps and memory for the N real number of additional memory. However, the gain for these minor costs is the ability to recalculate the incremental VaR impact value multiple times with minimal cost in time, as described below.

The DELVAR module 210 as described is one implementation for determining DELVAR. It is based on the following analysis of VAR.

Mathematically, VaR may be defined as follows: ##EQU1## where: P is a selected trading portfolio 130 of financial instruments 135 as described;

p=m (P) is a vector (column) of mapped cashflows 330, where m () is a mapping and shredding function provided by the map module 200 on the vertex set 175;

p' is the transpose of p;

Q is the covariance matrix 170 scaled by the square of the VaR probability standard deviations (which is typically 1.65.sup.2), the indices of the matrix also being the vertex set 175.

Assume that a candidate trade A.sub.(i) from the set of candidate trades 180 is to be made, and it is desired to determine the impact on the known VaR measure. The shredded and mapped cashflows of the candidate trade may be considered a vector a.sub.i, which is then scaled by the small positive quantity .epsilon.. As described above, a conventional approach determining the impact of this candidate trade A.sub.(i) on the VaR would be to shred and map candidate trade A.sub.(i) in the trading portfolio 130 to produce a revised map cashflow set r.sub.i. VaR would conventionally be recalculated as, where ##EQU2## where r.sub.i =p+.epsilon.a.sub.i. But a Taylor series expansion of VaR around .epsilon.=0 produces:

w.sub.i (.epsilon.)=w.sub.i (0)+.epsilon..gradient.w.sub.i (0).multidot.a.sub.i +o(.epsilon..sup.2) Eq.3

=v+.epsilon.(DELVAR.multidot.a.sub.i)+o(.epsilon..sup.2) Eq. 4

where .gradient. refers to the vector derivative operator, and where the vector index is the vertex set 175. The first term w.sub.i (0) is merely the original VaR. In Eq. 4, if .epsilon. is sufficiently small (and positive, since the candidate trade a.sub.i is being added in positive amount to the trading portfolio 130), then the improvement in the VaR measure is governed by the sign and magnitude of the second term of the Eq. 4. The higher order term (o(.epsilon..sup.2)) may be reasonably ignored, since .epsilon. is small. It is of course the case that most notational amounts of candidate trades 180 in an institution will be small relative to the size of the entire trading portfolio 130, justifying the approximation.

Direct calculation shows that: ##EQU3## Thus, the DELVAR vector depends only upon the selected trading portfolio 130, and not upon the selection of candidate trade A.sub.(i) from the set of candidate trades 180, even though the candidate trade A.sub.(i) was included in the conventional approach to the incremental determination. Thus, the same DELVAR vector works for all candidate trades 180, and does not itself have to be calculated more than once for the selected trading portfolio 180.

Returning then to FIG. 3, the incremental VaR impact value of each candidate trade 180 from the set of candidate trades is determined as follows:

The candidate trade 180 is shredded and mapped by the map module 200 on the vertex set 175 to produce its corresponding mapped cashflows 335. The mapped cashflows 335 are then optionally normalized by one of the normalization modules 220, as further described below. Whether or not the candidate trade 180 is normalized, the incremental VaR module 230 takes 312 the inner product of the mapped cashflows 335 and the DELVAR vector to produce the incremental VaR impact value. This determination of the incremental VaR impact value does not require recalculation of the VaR measure for the entire trading portfolio 130 as in conventional systems.

The incremental VaR impact value as described is useful for its sign value. If the incremental VaR impact value is negative, then the candidate trade 180 improves the VaR, and is thus a risk-reducing trade. If the sign is positive, the candidate trade 180 degrades the VaR by increasing portfolio risk. In this manner then, the trader can immediately determine, for each and every candidate trade 180 whether the trade is beneficial to the institution for risk reduction purposes.

This determination of the incremental impact on VaR of a candidate trade 180 may be expressed, in light of the foregoing, as:

›p'Q/v!.multidot.a.sub.i Eq. 6

The incremental VaR impact value as calculated via the DELVAR vector is an approximation, due to ignoring the higher order terms. However, because the typical trading portfolio 130 is large relative to the candidate trade 180, the incremental VaR impact value measured via the DELVAR vector is sufficiently quite accurate in most cases to provide the desired information for risk management.

The execution time of this inner product determination by the incremental VaR module 230 varies linearly with the number of vertices in the vertex set 175. The fact that the DELVAR vector is fixed for a selected trading portfolio 130 makes this linear computation time feasible, and is therefore a feature which provides a fast, incremental VaR calculation. This makes the financial analysis application 140 and the system 110 suitable for a real-time trading environment, even though it is an approximation to the true incremental VaR measure.

One benefit of the above approach and the determination of DELVAR vector for a trading portfolio 130 is the rapid identification, by the incremental VaR impact value, of those candidate trades which reduce or increase the VaR. However, because only the sign of the incremental VaR impact value

resulting from a candidate trade 180 might be ascertained, this approach alone does not provide means for comparing the relative worthiness of each of candidate trades 180. This is because any single candidate trade 180 may be arbitrarily doubled or halved in size, so that comparisons of magnitude of the incremental VaR impact value for different candidate trades 180 are meaningless. To accomplish comparability in risk, which is a goal of risk management, it is first desirable to assure that the candidate trades 180 are scaled in such a fashion as to become comparable with respect to a selected criterion. This step is termed herein "normalization". In one embodiment of the present invention, normalization is provided by selected one of the normalization modules 220.

In one embodiment of the present invention, there are six normalization modules 220, each providing a distinct criteria for comparing the incremental VaR impact value of candidate trades 180: cashflow normalization 220a, VaR normalization 220b, return normalization 220c, price normalization 220d, capital normalization 220e, and notional normalization 220f. Each normalization module 220 produces a normalization factor .lambda..sub.i calculated in a manner (detailed below) depending upon the candidate trade 180 being normalized. Reference is now made to FIG. 3, where calculation of the normalization factor 340 (notated "NF(i)" in FIG. 3) proceeds via one of several alternative methods:

1. Cashflow normalization 220a. In this module, a mathematical norm for the mapped cashflow 335 is associated with each candidate trade 180. If candidate trade a.sub.i consists of a vector of cashflows a.sub.i =(a.sub.i1, a.sub.i2, . . . , a.sub.in) then the normalization factor .lambda..sub.i (340) may be selected as one of: ##EQU4## where Eq. 7 normalizes the cashflow vector length, Eq. 8 uses the sum of the trade's absolute cashflows, and Eq. 9 employs the largest individual cashflow component. All of these may be further extended to an arbitrary positive weighting of the cashflows prior to normalization factor calculation, as shown by the weightings {c.sub.j } above.

2. VaR normalization 220b. In this module, the normalization is performed according to the VaR inherent in the candidate trade 180 itself. In effect, each candidate trade 180 is evaluated on the basis of equating the risk, measured via overall VaR measure, as if each candidate trade 180 were held in isolation. Accordingly, the normalization factor .lambda..sub.i (340) is calculated as ##EQU5## where Q is the covariance matrix 170. 3. Return normalization 220c. In this module, the normalization factor .lambda..sub.i (340) is selected according to the value of anticipated future returns accruing to holding the candidate trade 180. In one embodiment, this is determined by taking the net present value of all future revenues and payments of the candidate trade 180, as one such measure of future returns.

4. Price normalization 220d. In this module, the normalization factor .lambda..sub.i (340) is set equal to the market price of the candidate trade 180. This equates each candidate trade 180 according to their current mark-to-market, i.e., the value of the candidate trade 180 by present market standards.

5. Capital normalization 220e. In this module, the normalization factor .lambda..sub.i (340) is set equal to the regulatory or other amount of capital which must be allocated to sustain the candidate trade 180. For example, the Bureau of International Settlements (BIS) guidelines provide formulas involving certain capital adequacy underlying certain trade types.

6. Notional normalization 220f. In this module, a "notional value," of the candidate trade 180 is used, the notional value being an otherwise arbitrary market or other convention on the number of units involved in the candidate trade 180. For example, currency swap contracts are typically denominated in amounts involving $1 of principal payment, regardless of the swap interest rates involved. Because this normalization factor .lambda..sub.i (340) is completely arbitrary, it serves as a catch-all category of normalization, and may be used with any type of financial instrument 135.

For any of the normalization modules 220, then, the candidate trade cashflow vector 335 a.sub.i is normalized 345 as:

a.sub.i =›a.sub.i /.lambda..sub.i ! Eq. 11

The normalization modules 220 thus described may be implemented, in one embodiment, using pseudo-code such as:

```
SUBROUTINE NORMALIZE(A)
FOR I = 1 TO NUM.sub.-- CANDIDATES {
       LAMBDA = NORM(A, I, NUM.sub.-- VERTICES)
       FOR J = 1 TO NUM.sub.-- VERTICES DO {
         A>I,J! = A>I,J! / LAMBDA
       }
}
```

where the procedure NORM provides one of the six normalization factors 340 described above.

Accordingly, each of the candidate trades 180 is normalized 345 with one of the normalization modules 220 to produce for each candidate trade a normalized cashflow vector 350. This normalized cashflow 350 is then input into the incremental VaR module 230, and the incremental VaR impact value, VaR.sub.(i) for that candidate trade A.sub.(i) is determined 312, and stored.

Once all of the candidate trades 180 have been processed, and an incremental VaR impact value determined for each, these normalized candidate trades 350 are ranked 314 by their now-comparable incremental VaR impact value to produce a ranked set 355 of candidate trades. The trader risk manager, or officer can then review the ranked set 355 and select the candidate trade that best improves the VaR measure, in other words, the candidate trade that best reduces the value at risk for comparable values of the normalization. Preferably every such person is able to use the financial analysis application 140 for evaluating each candidate trade 180 throughout the trading period. This is done, as shown above, without having to recalculate the VaR measure for the trading portfolio 130 each time.

In summary, the present invention provides a system, a method, and a software product that beneficially determines the change in the VaR measure of a trading portfolio for any potential candidate trade that may be made and added to the portfolio. In addition, the present invention allows any variety of candidate trades to be compared for their impact on the VaR measure, thereby allowing a trader specifically, and a financial institution more generally, to identify and select those candidate trades that best reduce the VaR measure, and thereby improve the financial performance of the trading portfolio.

* * * * *

United States Patent 4,839,804
Roberts, et. al. Jun. 13, 1989

Method and apparatus for insuring the funding of a future liability of uncertain cost

Abstract

A method and apparatus are provided to insure a means of purchasing a floating rate zero coupon note that is designed to fund a certain future liability of uncertain value and thereby defease fully its future cost. The method is a one-year renewable term insurance program that fully funds the purchase of a certain floating rate zero coupon note upon the occurrence of some catastrophic event, such as the death of the insured. The system projects the expected death benefit payment and then calculates the annual insurance premium based on the expected death benefit payment, type of policy, and personal and risk characteristics of the insured.

Inventors: **Roberts; Peter A.** (New York, NY); **Finnerty; John D.** (New York, NY).
Assignee: **College Savings Bank** (Princeton, NJ).
Appl. No.: **947,614**
Filed: **Dec. 30, 1986**
Intl. Cl. : **G06F 15/21**
Current U.S. Cl.: **705/36**; 705/4
Field of Search: 364/400-402, 406, 408, 300, 715-716, 735

References Cited | [Referenced By]

U.S. Patent Documents

4,642,768	Feb., 1987	Roberts	364/408
4,722,055	Jan., 1988	Roberts	364/408

Primary Examiner: MacDonald; Allen
Attorney, Agent or Firm: Rogers; Laurence S., Ingerman; Jeffrey H.

66 Claims, 16 Drawing Figures

United States Patent **5,563,783**
Stolfo, et. al. **Oct. 8, 1996**

Method and system for securities pool allocation

Abstract

Trading in pooled securities (e.g., pooled mortgages) requires allocation of securities from pools to contracts subject to certain rules or constraints. To improve upon manual allocation procedures, computer techniques for fast and profitable allocation have been developed. Advantageously, a locally optimal allocation can be found by a rule-based greedy algorithm, and the locally optimal allocation can be improved upon further by a simulated annealing technique which is more likely to produce a globally optimal allocation.

Inventors: **Stolfo; Salvatore J.** (Ridgewood, NJ); **Yemini; Yechiam** (Briarcliff Manor, NY); **Pinsky; Eugene** (Brookline, MA).

Assignee: **The Trustees of Columbia University in the City of New York** (Morningside Heights, NY).

Appl. No.: **416,493**

Filed: **Apr. 4, 1995**

Related U.S. Application Data

Continuation of (including streamline cont.) Ser. No. 882,264, May 13, 1992, abandoned.

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Current U.S. Cl.:	**705/8**; 705/37
Field of Search:	364/406, 408, 648, 401

References Cited | [Referenced By]

U.S. Patent Documents

4,677,552	Jun., 1987	Sibley, Jr.	**364**/408
5,101,353	Mar., 1992	Lupien et al.	**364**/408

Other References

"The N-City Travelling Salesman Problem: Statistical Mechanics and the Metropolis Algorithm*" by Ernesto Bonomi and Jean-Luc Lutton, SIAM Review, vol. 26, No. 4, Oct. 1984.

"Concepts of Scale in Simulated Annealing" by Steve R. White, IBM Thomas J. Watson Research Center, 1984.

"Simulated Annealing: Theory and Applications" by P. J. M. van Laarhoven and E. H. L. Aarts, D. Reidel Publishing Company, Library of Congress Cataloging in Publication Data.

"Equation of State Calculations by Fast Computing Machines" by N. Metropolis, A. W. Rosenbluth, M. N. Rosenbluth, A. H. Teller and E. Teller. The Journal of Chemical Physics, vol. 21, No. 6, Jun. 1953.

"Optimization by Simulated Annealing" by S. Kirkpatrick, C. D. Gelatt, Jr., M. P. Vecchi. Science, May 13, 1983, vol. 220, No. 4598.

"The C Programming Language" by B. W. Kernighan and D. M. Ritchie. Prentice-Hall, Inc., 1978.

"Programming Expert Systems in OPS5" by L. Brownston, R. Farrell, E. Kant and N. Martin. Addison-Wesley Publishing Company, Inc., 1986.

"Mortgage Pass-through Securities: Ginnie Maes, Fannie Maes, and Freddie Macs" M. Stigum et al., The Dow Jones-Irwin Guide to Bond and Money Market Investments, Dow Jones-Irwin, 1987.

Primary Examiner: McElheny, Jr.; Donald E.
Assistant Examiner: Poinvil; Frantzy
Attorney, Agent or Firm: Brumbaugh, Graves, Donohue & Raymond

23 Claims, 2 Drawing Figures

(1 of 1)

United States Patent **5,471,575**
Giansante **Nov. 28, 1995**

Interactive parameter driven iterative financial spreadsheet analysis with context and layout sensitive help screen

Abstract

A financial analysis system for mathematically modeling an investment on a computer provides an improved computer user interface and a help method for teaching a user how to use the analysis system and for explaining the meaning of items on the screen. An investment term is broken up into two or more incremental periods and unique parameters may be associated with each incremental period so as to allow for the modeling of discrete events and permit varying of input assumptions of one or more incremental periods. In accordance with this scheme, results are displayed with each incremental period having its associated result or results. A method for displaying help text to explain the purpose, function and ordering of items displayed in a computer financial analysis system is disclosed. The help text is visually associated with an item that is displayed in context on the screen. Help text is automatically displayed in a predetermined sequence to explain items on the screen and to illustrate the logical flow of calculations performed on the items. Different help text may be displayed when the value of an item changes.

Inventors: **Giansante; Joseph E.** (Mountain View, CA).
Assignee: **Home Equity Software, Inc.** (Mountain View, CA).

Appl. No.: **816,892**
Filed: **Jan. 3, 1992**

Intl. Cl. : **G06F 17/21**
Current U.S. Cl.: **707/503**; 345/338
Field of Search: **395**/161, 144, 145, 146, 147, 148, 152, 153, 154, 155, 157, 158; **364**/408, 419.1

References Cited | [Referenced By]

U.S. Patent Documents

4,891,770	Jan., 1990	Hollett	**395**/144 X
4,989,141	Jan., 1991	Lyons et al.	**364**/408
4,992,939	Feb., 1991	Tyler	**364**/408 X
5,058,009	Oct., 1991	Yoshino et al.	**364**/408
5,122,972	Jun., 1992	Richards et al.	**395**/157
5,148,365	Sept., 1992	Dembo	**364**/408 X
5,155,806	Oct., 1992	Hoeber et al.	**395**/157
5,157,768	Oct., 1992	Hoeber et al.	**395**/157
5,179,654	Jan., 1993	Richards et al.	**395**/157
5,189,608	Feb., 1993	Lyons et al.	**364**/408

5,195,172	Mar., 1993	Elad et al.	**395/50**
5,287,448	Feb., 1994	Nichol et al.	**395/157 X**
5,317,688	May, 1994	Watson et al.	**395/161**
5,339,392	Aug., 1994	Risberg et al.	**395/161**

Other References

Campbell, Quattro Pro Handbook, 1990 pp. 6-7, 28-31, 66-67, 116-117, 144-145, 292-293, 334-337, 354-355, 376-381, 388-389, 543-547.

Albrecht, Teach Yourself GW Basic, 1990 pp. 18, 64.

Symantec, Timeline User Manual, 1990 pp. 16-14 & 16-15.

Mobley Matrix Intl, The Mobley Matrix Operations Manual, 1989 pp. P-1 to P-6, R-20, R-21.

Acerson, WordPerfect Series 5 The Complete Reference, 1988 pp. 231, 428.

WealthStarter with Charles J. Givens User's Guide. Reality Technologies, Limited.

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Assistant Examiner: Fetting; Anton W.
Attorney, Agent or Firm: Townsend and Townsend Khourie and Crew

8 Claims, 15 Drawing Figures

(1 of 1)

United States Patent	**5,148,365**
Dembo	**Sept. 15, 1992**

Scenario optimization

Abstract

A method and apparatus are provided for optimally allocating available resources in a physical system defined by a mathematical model having parameters of uncertain values. The method comprises the steps of firstly assigning a value to each of the uncertain parameters in the mathematical model based on a scenario that may or is expected to occur. Thereafter, given the parameter values at each possible scenario, the mathematical model is solved to yield the best solution of the mathematical model for that scenario. Once this has been complete, a probability value representing the expected probability that the scenario will occur is assigned to each scenario solution. The scenario parameter values, scenario solutions and scenario probabilities are then used to determine a single solution to the mathematical model which best "fits" the desired system behavior under the uncertainty defined by all of the scenarios considered. The single solution is then used to allocate the resources in the system. The present method is particularly useful in modelling a target portfolio from a number of other financial instruments.

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Appl. No.: **394,081**
Filed: **Aug. 15, 1989**

Intl. Cl. :	**G06F 15/20**
Current U.S. Cl.:	**705/36**
Field of Search:	**364**/402, 401, 408

References Cited | [Referenced By]

U.S. Patent Documents

4,346,442	Aug., 1982	Musmanno	
4,642,768	Feb., 1987	Roberts	
4,674,044	Jun., 1987	Kalmus et al.	
4,694,397	Sept., 1987	Grant et al.	
4,722,055	Jan., 1988	Roberts	
4,744,026	May, 1988	Vanderbei	
4,744,027	May, 1988	Bayer et al.	
4,744,028	May, 1988	Karmarkar	
4,752,877	Jun., 1988	Roberts et al.	
4,797,839	Jan., 1989	Powell	**364**/554
4,953,085	Aug., 1990	Atkins	**364**/408

Primary Examiner: Smith; Jerry
Assistant Examiner: Trammell; Jim
Attorney, Agent or Firm: Spencer, Frank & Schneider

18 Claims, 10 Drawing Figures

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PN/5222019: 1 patents.
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	Pat. No.	Title
1.	5,222,019	Financial calculator capable of displaying graphic representation

Search Summary

PN/5222019: 1 occurrence in 1 patent.

Search Time: 1.77 seconds.

United States Patent **5,222,019**
Yoshino, et. al. **Jun. 22, 1993**

Financial calculator capable of displaying graphic representation

Abstract

In a financial calculator, a graphic representation function is provided, and financial values desired for an operator are calculated based upon various input financial data. Also, both the entered data and calculated values are displayed as a graphic representation. In the graphic representation, a display interval of a pattern is determined based upon the number of the display data, and a magnitude of the pattern is determined based on the magnitude of the data value.

Inventors: **Yoshino; Hiroyuki** (Tokyo, JP); **Tomidokoro; Yoshinori** (Tokyo, JP).
Assignee: **Casio Computer Co., Ltd.** (Tokyo, JP).
Appl. No.: **770,634**
Filed: **Oct. 3, 1991**

Related U.S. Application Data

Division of Ser No. 292,379, Dec. 30, 1988.

Foreign Application Priority Data

Jan. 6, 1988 [JP]	63-818
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Intl. Cl. :	**G06F 15/30**
Current U.S. Cl.:	**705/36**
Field of Search:	**364/401, 408**

References Cited | [Referenced By]

U.S. Patent Documents

3,863,060	Jan., 1975	Rode et al.	**235/156**
3,946,218	Mar., 1976	Rode et al.	**235/156**
4,232,367	Nov., 1980	Youden et al.	**364/408**

Other References

Using Lotus 1.multidot.2.multidot.3, 1987, pp. 359-360.

"GMAC Mortgage Amortization Schedule".

"Ficales", Computer Language, vol. 4, No. 1, p.83, Jan. 1987, Tim Parker.

"HP Compound Interest and Loan Amortization", Product Literature, AN 83-003530 . . . File of Orbit.

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9 Claims, 27 Drawing Figures
